General Mathematics Unit 3 & 4 — Examination 1 (Practice 1) — Answers & Explanations

For a parent, tutor, or the student to mark the exam paper against.

Data analysis

1. C. 50.5

The interquartile range is 31 − 18 = 13. The upper fence is Q3 + 1.5 × IQR = 31 + 19.5 = 50.5, so any value above 50.5 is an outlier.

2. A. 14

For Store B the interquartile range is Q3 − Q1 = 38 − 24 = 14. Store A has the smaller interquartile range, 40 − 28 = 12, so the middle 50% of Store A's sales is less spread out.

3. E. categorical and nominal

A postcode is written with digits, but the digits label an area rather than measure anything — postcode 3000 is not "less than" postcode 3121 in any useful sense, and averaging postcodes is meaningless. Because the categories have no natural order, the variable is nominal.

4. B. 95%

The values 62 and 78 are each two standard deviations from the mean, because 70 − 2 × 4 = 62 and 70 + 2 × 4 = 78. By the 68–95–99.7% rule, 95% of values lie within two standard deviations of the mean.

5. D. −1.5

The standardised score is z = (x − mean) ÷ standard deviation = (60 − 72) ÷ 8 = −1.5. The score is negative because 60 is below the mean.

6. B. 79

Rearranging z = (x − mean) ÷ standard deviation gives x = mean + z × standard deviation = 64 + 1.25 × 12 = 79.

7. C. 5

With 13 values, the median is the 7th value when they are ordered. Counting along the dot plot, the 7th value is 5. The mode is 6, which occurs four times, but the question asks for the median.

8. A. 43.8%

There are 35 + 45 = 80 females, of whom 35 answered Yes. The percentage is 35 ÷ 80 = 0.4375, which is 43.8%. The value 45.5% is the percentage of Yes responses that came from females, a different question.

9. E. 46%

The coefficient of determination is $r^2 = (-0.68)^2 = 0.4624$, which is 46% correct to the nearest per cent. The sign of r says the association is negative but does not affect r^2.

10. C. each extra kilometre adds $3.20 to the cost

The slope is the change in the response variable for a one-unit increase in the explanatory variable. Here a one-kilometre increase in distance is associated with a $3.20 increase in cost, on average. The intercept, $12.50, is the fixed cost of a delivery of zero distance.

11. A. −2.5

The predicted value is y = 8.4 + 1.6 × 12 = 27.6. The residual is the actual value minus the predicted value: 25.1 − 27.6 = −2.5. The residual is negative because the line overpredicts at this point.

12. C. the association is not linear, so a linear model is not appropriate

A residual plot is used to check the assumption of linearity. Random scatter about zero supports a linear model; a clear pattern such as a curve means the relationship is non-linear, and the data should be transformed or a different model used.

13. B. a $\log_{10}(x)$ transformation

The pattern rises but flattens, so the x-values need to be stretched at the low end and compressed at the high end. A $\log_{10}(x)$ transformation does exactly that. An x^2 transformation would exaggerate the curve rather than remove it.

14. D. 30

The three-point moving average for month 4 uses months 3, 4 and 5: (26 + 30 + 34) ÷ 3 = 90 ÷ 3 = 30. Only the value either side of the month is used, not the whole series.

15. E. $6400

Deseasonalised figure = actual figure ÷ seasonal index = 8000 ÷ 1.25 = 6400. An index above 1 means winter is a busier-than-average season, so the deseasonalised value is lower than the actual value.

16. A. 0.85

Seasonal indices for a year must average 1, so for four seasons they sum to 4. The three given indices sum to 1.15 + 0.92 + 1.08 = 3.15, so the spring index is 4 − 3.15 = 0.85.

Recursion and financial modelling

17. D. 76

Applying the rule twice: $A_1 = 3 \times 12 - 8 = 28$, then $A_2 = 3 \times 28 - 8 = 76$. The value 28 is A_1, one step short.

18. B. $9600

Flat rate depreciation is a fixed amount each year, 0.15 × 24 000 = $3600. Over four years the machine loses 4 × 3600 = $14 400, leaving 24 000 − 14 400 = $9600. Reducing balance depreciation at the same rate would instead leave about $12 528.

19. E. $V_0 = 32\,000$, $V_{n+1} = 0.88V_n$

Losing 12% of its value each year leaves 88% of the previous year's value, so $V_{n+1} = 0.88V_n$. This is reducing balance depreciation. Subtracting a fixed $3840 each year would be flat rate depreciation instead.

20. D. $30 000

Unit cost depreciation charges a fixed amount per unit of use: 0.25 × 60 000 = $15 000 of depreciation. The value is 45 000 − 15 000 = $30 000.

21. A. $6927.31

The monthly interest rate is 4.8 ÷ 12 = 0.4%, and three years is 36 compounding periods. The value is 6000 × 1.004^{36} = $6927.31. Simple interest at the same rate would give only $6864.00.

22. D. 4.2%

The multiplier 1.0035 means 0.35% interest is charged each month. The annual rate is 12 × 0.35 = 4.2%. The monthly repayment of $1250 does not affect the interest rate.

23. B. $475

The monthly interest rate is 5.4 ÷ 12 = 0.45%, so the first month's interest is 250 000 × 0.0045 = $1125. The rest of the repayment reduces the principal: 1600 − 1125 = $475.

24. E. 4.5%

A perpetuity pays out exactly the interest earned, leaving the principal untouched. The monthly rate is 450 ÷ 120 000 = 0.00375, which is 0.375% per month, so the annual rate is 12 × 0.375 = 4.5%.

Matrices

25. C. 8

The element m_{ij} is in row i and column j, so m_{23} is in row 2, column 3. Reading across the second row, 9, 3, 8, the third entry is 8. The value 2 is m_{13}, from the wrong row.

26. A. 3 × 4

A product is defined when the number of columns of the first matrix equals the number of rows of the second: A has 2 columns and B has 2 rows, so AB exists. The product takes the rows of A and the columns of B, giving order 3 × 4.

27. E. 20

The element in row 1, column 2 of AB comes from row 1 of A and column 2 of B: 2 × 1 + 3 × 6 = 2 + 18 = 20. The value 16 is the element in row 1, column 1.

28. B. 10

For a 2 × 2 matrix with first row a, b and second row c, d, the determinant is ad − bc. Here 6 × 3 − 4 × 2 = 18 − 8 = 10. Because the determinant is not zero, P has an inverse.

29. D. 550

Starting with 500 in each state, the number in state A after one week is 0.8 × 500 + 0.3 × 500 = 400 + 150 = 550. The remaining 450 are in state B, and the two still total 1000.

30. C. 600

In the steady state the numbers stop changing, so 0.9a + 0.2b = a, which gives 0.2b = 0.1a and therefore a = 2b. Since a + b = 900, we get 3b = 900, so b = 300 and a = 600. The long-run split is two-thirds in state A.

31. A. reorder the elements of the column matrix

Each row of a permutation matrix picks out exactly one element of the column matrix and leaves the others out, so the result contains the same values rearranged. The order of the matrix is unchanged, and only the identity matrix leaves the values in place as well.

32. B. 2

Rearranging gives 3X = D − C. In row 2, column 1 that is 7 − 1 = 6, so the element of X is 6 ÷ 3 = 2. The value 6 is the corresponding element of 3X, before dividing.

Networks and decision mathematics

33. C. 12

Every edge contributes 1 to the degree of each of its two endpoints, so the sum of the degrees is always twice the number of edges. This graph has 6 edges, so the sum of the degrees is 12.

34. E. 5

Euler's formula for a connected planar graph is v + f = e + 2. Substituting gives 6 + f = 9 + 2, so f = 5. The faces include the infinite region outside the graph.

35. A. 19

Adding edges from lightest to heaviest and skipping any that would form a cycle gives BC (3), AB (4), CD (5) and then DE (7); AC (6) is skipped because it would close a cycle. A spanning tree on 5 vertices has 4 edges, and their total weight is 3 + 4 + 5 + 7 = 19.

36. D. 14

The path P–Q–R–S–T has length 5 + 3 + 2 + 4 = 14. Every other route is longer: P–R–S–T is 15, P–Q–S–T is 16 and P–R–T is 17.

37. B. 18

Assigning Wendy to Task 2, Xavier to Task 1 and Yuki to Task 3 gives 7 + 6 + 5 = 18 minutes. Every other allocation of one worker per task takes longer — the next best is 20 minutes.

38. E. 16

The project cannot finish before its longest path is complete. That path is A–C–F–G, taking 4 + 5 + 3 + 4 = 16 days. The other paths are shorter: A–D–F–G takes 13 days and B–E–G takes 13 days.

39. C. 16

The cut separating T from the rest of the network has capacity 7 + 9 = 16, and no cut is smaller, so by the maximum-flow minimum-cut theorem the maximum flow is 16. This flow is achievable: send 7 along A to T and 9 along B to T, with B supplied by 6 from S and 3 from A.

40. D. 6

A tree is a connected graph with no cycles, and a tree on n vertices always has exactly n − 1 edges. With 7 vertices that is 6 edges. Adding any further edge would create a cycle.