Maths Methods Unit 3 & 4 — Examination 1 (Practice 1) — Answers & Explanations

For a parent, tutor, or the student to mark the exam paper against.

Examination 1 — technology-free

1. (4 marks in total)

a. $dy/dx = xe^{3x}(2 + 3x)$ [2 marks]

Product rule with $u = x^2$ and $v = e^{3x}$: $dy/dx = 2xe^{3x} + 3x^2e^{3x}$. One mark for applying the product rule, one for the simplified factorised form $xe^{3x}(2 + 3x)$.

b. $f(x) = (2x - 3)^5/10 + 9/10$ [2 marks]

Antidifferentiate: $f(x) = (2x - 3)^5/10 + c$ (one mark). Substituting $f(2) = 1$ gives $1/10 + c = 1$, so $c = 9/10$ (one mark).

2. (3 marks in total)

a. $a = 2$ [1 mark]

The vertical asymptote of $y = m/(x - a) + c$ is $x = a$, and the asymptote is given as $x = 2$.

b. $m = -18$ and $c = -6$ [2 marks]

The y-intercept gives $-m/2 + c = 3$ and the x-intercept gives $m/(-3) + c = 0$, so $c = m/3$. Substituting: $-m/2 + m/3 = 3$, so $-m/6 = 3$ and $m = -18$, giving $c = -6$. One mark for setting up both equations, one for solving.

3. (5 marks in total)

a. $k = 0.4$ [1 mark]

The probabilities must sum to 1: $0.1 + 0.3 + k + 0.2 = 1$, so $k = 0.4$.

b. $E(X) = 1.7$ [2 marks]

$E(X) = 0(0.1) + 1(0.3) + 2(0.4) + 3(0.2) = 0.3 + 0.8 + 0.6 = 1.7$. One mark for the correct sum of products, one for the value.

c. $2/3$ [2 marks]

$\Pr(X \geq 2) = 0.4 + 0.2 = 0.6$ and $\Pr(X \geq 1) = 0.3 + 0.4 + 0.2 = 0.9$. Since $\{X \geq 2\}$ is contained in $\{X \geq 1\}$, the conditional probability is $0.6/0.9 = 2/3$. One mark for each probability, or one for the correct quotient set up.

4. (4 marks in total)

a. $2\pi/3$ [1 mark]

The period of $y = a\cdot\sin(nx)$ is $2\pi/n$, and here $n = 3$.

b. $x = \pi/9, 2\pi/9, 7\pi/9, 8\pi/9$ [3 marks]

$\sin(3x) = \sqrt{3}/2$. For $x \in [0, \pi]$ the argument $3x$ ranges over $[0, 3\pi]$, where $\sin\theta = \sqrt{3}/2$ has solutions $\theta = \pi/3, 2\pi/3, 7\pi/3$ and $8\pi/3$. Dividing each by 3 gives the four values. One mark for $\sin(3x) = \sqrt{3}/2$, one for the correct domain for $3x$, one for all four solutions.

5. (5 marks in total)

a. $x = 0$ and $x = 2$ [2 marks]

Factorise: $3x(x - 2) = 0$. One mark for the factorisation, one for both intercepts.

b. 4 square units [3 marks]

The graph lies below the axis between the intercepts, so the area is the absolute value of the integral. $\int_0^2(3x^2 - 6x)dx = [x^3 - 3x^2]_0^2 = (8 - 12) - 0 = -4$, giving an area of 4. One mark for the correct integral and terminals, one for evaluating it, one for taking the absolute value.

6. (4 marks in total)

a. $f^{-1}(x) = e^{(x/2)} + 1$ [2 marks]

Swap and solve: $x = 2\log_e(y - 1)$, so $x/2 = \log_e(y - 1)$ and $y = e^{(x/2)} + 1$. One mark for isolating the logarithm, one for the correct rule.

b. Domain R, range $(1, \infty)$ [2 marks]

The domain of the inverse is the range of f, which is R, and the range of the inverse is the domain of f, which is $(1, \infty)$. One mark each.

7. (5 marks in total)

a. $y = -x - 2$ [2 marks]

$f(1) = -3$ and $f'(x) = 3x^2 - 4$, so $f'(1) = -1$. The tangent is $y + 3 = -1(x - 1)$, that is $y = -x - 2$. One mark for the gradient, one for the equation.

b. $(-2, 0)$ [3 marks]

Solve $x^3 - 4x = -x - 2$, giving $x^3 - 3x + 2 = 0$. Since $x = 1$ is a known solution it factorises as $(x - 1)^2(x + 2) = 0$, so the other solution is $x = -2$, where $y = 0$. One mark for forming the equation, one for the factorisation, one for the coordinates.

8. (5 marks in total)

a. $x = 0$ and $x = 2$ [2 marks]

Let $u = 2^x$, giving $u^2 - 5u + 4 = 0$ and $(u - 1)(u - 4) = 0$, so $u = 1$ or $u = 4$. Then $2^x = 1$ gives $x = 0$ and $2^x = 4$ gives $x = 2$. One mark for the substitution, one for both solutions.

b. $x = 4$ [3 marks]

Combine to $\log_e(x(x - 3)) = \log_e(4)$, so $x^2 - 3x - 4 = 0$ and $(x - 4)(x + 1) = 0$. Both logarithms require $x > 3$, so $x = -1$ is rejected and $x = 4$ is the only solution. One mark for combining, one for solving the quadratic, one for rejecting $x = -1$ on domain grounds.

9. (5 marks in total)

a. $k = 1/8$ [2 marks]

The total area under a probability density function is 1: $\int_0^4 kx\,dx = k[x^2/2]_0^4 = 8k = 1$, so $k = 1/8$. One mark for setting the integral equal to 1, one for the value.

b. $1/4$ [3 marks]

$\Pr(X \le 2) = \int_0^2 (1/8)x\,dx = (1/8)[x^2/2]_0^2 = (1/8)(2) = 1/4$. One mark for the correct integral, one for the terminals, one for the value.