Mathematical Methods Unit 3 & 4 — Examination 1 (Practice 3) — Answers & Explanations

Answers to the 5 questions in the free preview.

Examination 1 — technology-free

1. (4 marks in total)

a. $dy/dx = 6x/(3x^2 + 1)$ [1 mark]

Chain rule: the derivative of $\log_e(u)$ is u'/u, with $u = 3x^2 + 1$ and $u' = 6x$.

b. $f'(\pi/4) = 1 + \pi/2$ [3 marks]

Product rule: $f'(x) = \tan(x) + x\sec^2(x) = \tan(x) + x/\cos^2(x)$. At $x = \pi/4$, $\tan(\pi/4) = 1$ and $\cos^2(\pi/4) = 1/2$, so $f'(\pi/4) = 1 + (\pi/4)(2) = 1 + \pi/2$. One mark for the product rule, one for the derivative of $\tan(x)$, one for the value.

2. (4 marks in total)

a. $-(1/2)e^{-2x} - (1/3)\cos(3x) + c$ [2 marks]

$\int e^{-2x}\,dx = -(1/2)e^{-2x}$ and $\int \sin(3x)\,dx = -(1/3)\cos(3x)$. One mark for each term; the constant of integration is expected.

b. 3/2 [2 marks]

An antiderivative is $e^x - e^{-x}$. At $x = \log_e(2)$ this is $2 - 1/2 = 3/2$, and at $x = 0$ it is $1 - 1 = 0$. The value is 3/2. One mark for the antiderivative, one for the value.

3. (4 marks in total)

a. The two rules agree at $x = 1$: $1 + a = b + 1$, so $a = b$. [1 mark]

For continuity the left-hand limit $1^2 + a$ must equal $f(1) = b + 1$, which gives $a = b$.

b. $a = 2$, $b = 2$ [2 marks]

The gradient of $x^2 + a$ at $x = 1$ is $2x = 2$, and the gradient of $bx + 1$ is b. Differentiability requires the gradients to match, so $b = 2$, and continuity then gives $a = 2$. One mark for equating the derivatives, one for both values.

c. $[2, \infty)$ [1 mark]

For $x < 1$, $x^2 + 2$ takes every value in $[2, \infty)$ (the minimum is 2 at $x = 0$). For $x \geq 1$, $2x + 1$ takes values in $[3, \infty)$. The range is $[2, \infty)$.

4. (5 marks in total)

a. 0.2 [1 mark]

$\Pr(A \cap B) = \Pr(A) + \Pr(B) - \Pr(A \cup B) = 0.5 + 0.3 - 0.6 = 0.2$.

b. 2/3 [1 mark]

$\Pr(A \mid B) = \Pr(A \cap B)/\Pr(B) = 0.2/0.3 = 2/3$.

c. Not independent, because $\Pr(A) \times \Pr(B) = 0.15 \neq 0.2 = \Pr(A \cap B)$. [1 mark]

Independence requires $\Pr(A \cap B) = \Pr(A)\Pr(B)$. Equivalently, $\Pr(A \mid B) = 2/3$ differs from $\Pr(A) = 0.5$.

d. 1/2 [2 marks]

$\Pr(A \cap B') = \Pr(A) - \Pr(A \cap B) = 0.3$. The event $A \cap B'$ lies inside $A \cup B$, so the conditional probability is $0.3/0.6 = 1/2$. One mark for $\Pr(A \cap B')$, one for the answer.

5. (4 marks in total)

a. $x = 7/3$ [2 marks]

$\log_2((x + 3)/(x - 1)) = 2$, so $(x + 3)/(x - 1) = 4$ and $x + 3 = 4x - 4$, giving $x = 7/3$. This satisfies $x > 1$, so both logarithms are defined. One mark for combining the logarithms, one for the solution.

b. $x = \log_e(2)/\log_e(3/2)$ [2 marks]

Take natural logarithms: $(x + 1)\log_e(2) = x\log_e(3)$, so $x(\log_e(3) - \log_e(2)) = \log_e(2)$ and $x = \log_e(2)/\log_e(3/2)$. One mark for taking logarithms, one for the answer.

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PREVIEW

That’s the end of the preview answers

These are the answers to the 5 preview questions. The full answer key covers all 9 questions in Mathematical Methods Unit 3 & 4 — Examination 1 (Practice 3), with the working behind every mark.

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