Mathematical Methods Unit 3 & 4 — Examination 1 (Practice 4) — Answers & Explanations

Answers to the 5 questions in the free preview.

Examination 1 — technology-free

1. (4 marks in total)

a. $dy/dx = 12(3x - 1)^3$ [1 mark]

Chain rule: $4(3x - 1)^3 \times 3$.

b. $f'(e) = 3e$ [2 marks]

Product rule: $f'(x) = 2x\log_e(x) + x^2 \times (1/x) = 2x\log_e(x) + x$. At $x = e$: $2e \times 1 + e = 3e$. One mark for the derivative, one for the value.

c. $g'(\pi) = -1$ [1 mark]

g′(x) = cos(x)e^(sin(x)). At $x = \pi$: $\cos(\pi)e^0 = -1$.

2. (3 marks in total)

a. $2e^{2x} - 3\log_e(x) + c$ [2 marks]

One mark for each term. The constant of integration is expected.

b. f(x) = 2x^(3/2) + 4 [1 mark]

An antiderivative of 3x^(1/2) is 2x^(3/2). Then $f(4) = 2 \times 8 + c = 20$ gives $c = 4$.

3. (5 marks in total)

a. $x = -1$ and $y = 2$ [1 mark]

The vertical asymptote is where $x + 1 = 0$; the horizontal asymptote is the constant term.

b. $(0, -1)$ and $(1/2, 0)$ [2 marks]

$f(0) = 2 - 3 = -1$. For the x-intercept, $3/(x + 1) = 2$ gives $x + 1 = 3/2$, so $x = 1/2$. One mark for each intercept.

c. $f^{-1}(x) = 3/(2 - x) - 1$, domain $R \setminus \{2\}$ [2 marks]

Swap x and y: $x = 2 - 3/(y + 1)$, so $3/(y + 1) = 2 - x$ and $y = 3/(2 - x) - 1$. The domain of f^{-1} is the range of f, $R \setminus \{2\}$. One mark for the rule, one for the domain.

4. (4 marks in total)

a. $[-1, 3]$ [1 mark]

cos(x) takes every value in $[-1, 1]$ on this domain, so $1 - 2\cos(x)$ runs from $1 - 2 = -1$ to $1 + 2 = 3$.

b. $x = 2\pi/3$ or $x = 4\pi/3$ [2 marks]

$1 - 2\cos(x) = 2$ gives $\cos(x) = -1/2$, so $x = \pi - \pi/3 = 2\pi/3$ or $x = \pi + \pi/3 = 4\pi/3$. One mark for each solution.

c. $x = \pi/3$ and $x = 5\pi/3$ [1 mark]

$\cos(x) = 1/2$ gives $x = \pi/3$ or $x = 2\pi - \pi/3 = 5\pi/3$.

5. (5 marks in total)

a. $Pr(X = 3) = 1/6$, $Pr(X = 0) = 1/3$, $Pr(X = -2) = 1/2$ [1 mark]

A six gives $5 - 2 = 3$; a four or five gives $2 - 2 = 0$; otherwise the player loses the $2.

b. $-1/2$ [1 mark]

$E(X) = 3 \times 1/6 + 0 \times 1/3 - 2 \times 1/2 = 1/2 - 1 = -1/2$.

c. 13/4 [2 marks]

$E(X^2) = 9 \times 1/6 + 0 + 4 \times 1/2 = 3/2 + 2 = 7/2$. $Var(X) = 7/2 - (-1/2)^2 = 7/2 - 1/4 = 13/4$. One mark for $E(X^2)$, one for the variance.

d. 11/36 [1 mark]

Positive totals come from (3, 3), (3, 0), (0, 3), (3, −2) and (−2, 3): $1/36 + 2 \times 1/18 + 2 \times 1/12 = 1/36 + 4/36 + 6/36 = 11/36$.

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PREVIEW

That’s the end of the preview answers

These are the answers to the 5 preview questions. The full answer key covers all 9 questions in Mathematical Methods Unit 3 & 4 — Examination 1 (Practice 4), with the working behind every mark.

STILL TO COME	QUESTIONS	MARKS
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The full paper comes with

- Every question, printed exactly like the ones in this preview
- The complete answer key, with the working behind every mark
- A topic report after marking, showing what to practice next

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