Mathematical Methods Unit 3 & 4 — Examination 1 (Practice 5) — Answers & Explanations

Answers to the 5 questions in the free preview.

Examination 1 — technology-free

1. (4 marks in total)

a. $f'(x) = ((x^2 + 1) - x \times 2x)/(x^2 + 1)^2 = (1 - x^2)/(x^2 + 1)^2$ [2 marks]

One mark for applying the quotient rule, one for simplifying.

b. Local maximum (1, 1/2); local minimum (−1, −1/2) [2 marks]

$f'(x) = 0$ when $x = \pm 1$. f' changes from positive to negative at $x = 1$ and from negative to positive at $x = -1$. One mark for the coordinates, one for the nature.

2. (4 marks in total)

a. $-(1/2)\cos(2x) + (1/2)\log_e(2x + 1) + c$ [2 marks]

One mark for each term. The constant of integration is expected.

b. 1/3 [1 mark]

$[(1/3)\sin(3x)]$ from 0 to $\pi/6 = (1/3)\sin(\pi/2) - 0 = 1/3$.

c. $x\log_e(x) - x$ [1 mark]

Integrating both sides gives $x\log_e(x) = \int \log_e(x)\,dx + x$, so $\int \log_e(x)\,dx = x\log_e(x) - x + c$.

3. (5 marks in total)

a. $x = 1$ and $y = -1$ [1 mark]

The vertical asymptote is where $x - 1 = 0$; the horizontal asymptote is the constant term.

b. (0, 2), $(1 - \sqrt{3}, 0)$ and $(1 + \sqrt{3}, 0)$ [3 marks]

$f(0) = 3 - 1 = 2$. For the x-intercepts, $(x - 1)^2 = 3$, so $x = 1 \pm \sqrt{3}$. One mark for the y-intercept, two for the x-intercepts.

c. $(-1, \infty)$ [1 mark]

$3/(x - 1)^2$ takes every positive value but never 0.

4. (5 marks in total)

a. Period 4; range $[-2, 4]$ [2 marks]

The period is $2\pi \div (\pi/2) = 4$. The amplitude is 3 about the line $y = 1$. One mark each.

b. $x = 1/3$ or $x = 5/3$ [2 marks]

$\sin(\pi x/2) = 1/2$ gives $\pi x/2 = \pi/6$ or $5\pi/6$. One mark for each solution.

c. 5 [1 mark]

$\sin(\pi x/2) = 0$ when $x = 0, 2, 4, 6$ or 8.

5. (4 marks in total)

a. $x = 1$ or $x = 2$ [2 marks]

Let $u = 2^x$. Then $2^{x+1} = 2u$ and the equation is $u^2 - 6u + 8 = 0$, so $u = 2$ or $u = 4$. One mark for the quadratic in u, one for both solutions.

b. $x = 4$ [2 marks]

$x(x - 3) = 4$ gives $x^2 - 3x - 4 = (x - 4)(x + 1) = 0$. $x = -1$ is rejected because $\log_e(x)$ is undefined there. One mark for the quadratic, one for rejecting $x = -1$.

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PREVIEW

That’s the end of the preview answers

These are the answers to the 5 preview questions. The full answer key covers all 9 questions in Mathematical Methods Unit 3 & 4 — Examination 1 (Practice 5), with the working behind every mark.

STILL TO COME	QUESTIONS	MARKS
Examination 1 — technology-free	4	18

The full paper comes with

- Every question, printed exactly like the ones in this preview
- The complete answer key, with the working behind every mark
- A topic report after marking, showing what to practice next

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