

EXAM PAPER

Specialist Mathematics Unit 3 & 4 — Examination 1 (Practice 1)

Reading time: 15 minutes (no writing)

Writing time: 60 minutes

Total time: 75 minutes

SECTIONS IN THIS PAPER

- Examination 1 — technology-free — 60 min (no calculator)

INSTRUCTIONS

You are not permitted to write during reading time — you may only read the paper and plan your approach. Answer every question you can. Write your working in the space provided for long-answer questions. Marking guidance and full explanations are provided in the separate answer key.

Examination 1 — technology-free

60 minutes • No calculator

Question 1

(3 marks)

Prove by mathematical induction that $1 + 3 + 5 + \dots + (2n - 1) = n^2$ for all $n \in \mathbb{N}$.

- a. Show that the statement is true for $n = 1$.

1 mark

- b. Assume the statement is true for $n = k$. Show that it is then true for $n = k + 1$, and hence complete the proof.

2 marks

Question 2

(4 marks)

Let $z = 1 - \sqrt{3}i$.

- a. Write z in polar form.

1 mark

- b. Find z^6 .

1 mark

- c. Find the square roots of z , in polar form.

2 marks

Question 3

(5 marks)

Let $f(x) = (x^2 + 1)/(x - 1)$, $x \neq 1$.

- a. Express $f(x)$ in the form $x + a + b/(x - 1)$, where a and b are integers.

1 mark

- b. Write down the equations of the asymptotes of the graph of f .

2 marks

- c. Find the coordinates of the stationary points of the graph of f . 2 marks

Question 4 (4 marks)

This question is about integration techniques.

- a. Using the substitution $u = \cos(x)$, evaluate $\int \sin(x)\cos^3(x) dx$ from $x = 0$ to $x = \pi/2$. 2 marks

- b. Use integration by parts to find $\int xe^{2x} dx$. 2 marks

Question 5 (4 marks)

Consider the differential equation $dy/dx = 2xy$, where $y(0) = 3$.

- a. Solve the differential equation, expressing y in terms of x . 3 marks

- b. Find y when $x = \sqrt{(\log_e(2))}$. 1 mark

Question 6 (4 marks)

Let $a = 2i - j + 2k$ and $b = i + 2j - 2k$.

- a. Find $a \cdot b$. 1 mark

- b. Find the angle between a and b , in terms of an inverse circular function. 1 mark

c. Find $a \times b$.

2 marks

Question 7

(4 marks)

The independent random variables X and Y have $E(X) = 5$, $\text{Var}(X) = 4$, $E(Y) = 3$ and $\text{Var}(Y) = 9$.

a. Find $E(2X - 3Y)$.

1 mark

b. Find $\text{Var}(2X - 3Y)$.

2 marks

c. Random samples of 16 values of X are taken. Find the standard deviation of the sample mean $\bar{X}$.

1 mark

Question 8

(4 marks)

The curve with equation $x^2 + xy + y^2 = 7$ passes through the point $(1, 2)$.

a. Use implicit differentiation to find dy/dx at the point $(1, 2)$.

3 marks

b. Find the equation of the tangent to the curve at $(1, 2)$.

1 mark

Question 9

(4 marks)

Let $P(z) = z^3 - 2z^2 + 9z - 18$, where $z \in \mathbb{C}$.

a. Show that $z - 2$ is a factor of $P(z)$.

1 mark

b. Solve $P(z) = 0$ for $z \in \mathbb{C}$.

2 marks

c. The roots of $P(z) = 0$ are plotted in the complex plane. Find the area of the triangle with these points as vertices.

1 mark

Question 10

(4 marks)

This question is about areas and volumes.

a. The region bounded by the graph of $y = \sqrt{x}$, the x-axis and the line $x = 4$ is rotated about the x-axis. Find the volume of the solid formed.

2 marks

b. Find the area of the region bounded by the graph of $y = 1/(1 + x^2)$, the x-axis and the lines $x = 0$ and $x = 1$.

2 marks
