Specialist Mathematics Unit 3 & 4 — Examination 2 (Practice 1) — Answers & Explanations

Answers to the 13 questions in the free preview.

Section A — multiple choice

1. B. There exists a real number x such that $x > 2$ and $x^2 \leq 4$.

The negation of "for all x, $P(x) \Rightarrow Q(x)$" is "there exists x with $P(x)$ and not $Q(x)$". A negates only the conclusion and keeps "for all"; D is the contrapositive, which is equivalent to the original statement, not its negation.

2. C. 6

A counterexample makes the hypothesis true and the conclusion false. $6^2 = 36$ is divisible by 4 but 6 is not. For $n = 3$ the hypothesis is false; for 4 and 8 the conclusion is true.

3. A. a vertical asymptote $x = -1$ and a horizontal asymptote $y = 1$ only

$\dfrac{x^2-4}{x^2-x-2} = \dfrac{(x-2)(x+2)}{(x-2)(x+1)} = \dfrac{x+2}{x+1}$, $x \neq 2$. The factor $x - 2$ cancels, so $x = 2$ is a point of discontinuity (a hole at $\left(2, \frac{4}{3}\right)$), not an asymptote. The only vertical asymptote is $x = -1$, and $y \to 1$ as $x \to \pm\infty$.

4. D. $\left[0, \frac{2}{3}\right]$ and $\left[-\frac{\pi}{2}, \frac{3\pi}{2}\right]$

The domain needs $-1 \leq 1 - 3x \leq 1$, so $0 \leq x \leq \frac{2}{3}$. $\sin^{-1}$ has range $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$; doubling gives $[-\pi, \pi]$ and adding $\frac{\pi}{2}$ gives $\left[-\frac{\pi}{2}, \frac{3\pi}{2}\right]$. Option C omits the translation.

5. A. $\dfrac{5\pi}{12}$

$\operatorname{Arg}(z) = -\frac{\pi}{4}$ and $\operatorname{Arg}(w) = \frac{\pi}{6}$, so an argument of $\frac{z^5}{w^2}$ is $5\left(-\frac{\pi}{4}\right) - 2\left(\frac{\pi}{6}\right) = -\frac{19\pi}{12}$. This is outside $(-\pi, \pi]$; adding 2π gives the principal argument $\frac{5\pi}{12}$.

6. A. $\{z : 1 \leq |z| \leq 2\} \cap \left\{z : 0 \leq \operatorname{Arg}(z) \leq \frac{\pi}{3}\right\}$

The region lies between the circles of radius 1 and 2 centred at the origin. Its upper edge is the ray through $(1, \sqrt{3})$, which makes an angle $\tan^{-1}(\sqrt{3}) = \frac{\pi}{3}$ with the positive real axis, and its lower edge is the positive real axis. C would include the points inside the unit circle; D describes circles centred at 1.

7. B. -6

The coefficients are real, so $2 - i$ is also a root. The product of the roots of $z^3 + bz^2 + cz + d$ is $-d = 10$, and $(2 + i)(2 - i) = 5$, so the third root is 2. The sum of the roots is $-b = 6$, so $b = -6$. ($c = 13$ is the sum of the products of pairs.)

8. C. 1.362

Each pass computes $y \leftarrow y + 0.1(x + y)$ and then $x \leftarrow x + 0.1$: $y_1 = 1 + 0.1(0 + 1) = 1.1$, $y_2 = 1.1 + 0.1(0.1 + 1.1) = 1.22$, $y_3 = 1.22 + 0.1(0.2 + 1.22) = 1.362$. 1.220 stops after two passes; 1.331 ignores x in f; 1.395 updates x before y.

9. D. $\dfrac{dy}{dx} = x - y$

Along the line $y = x$ the segments are horizontal, so $\frac{dy}{dx} = 0$ when $y = x$; this rules out A and B. At $(1, 0)$ the gradient is positive, so the rule is $x - y$ (C gives -1 there).

10. C. 2.27

$\dfrac{dv}{dt} = 2 - v$ with $v(0) = 0$ gives $v = 2(1 - e^{-t})$. The distance is $\displaystyle\int_0^2 2(1 - e^{-t})\,dt = 2 + 2e^{-2} \approx 2.27$. 1.73 is the velocity at $t = 2$; 4.00 assumes the limiting velocity 2 m s⁻¹ throughout.

11. B. 8π

About the y-axis the radius is $x = \sqrt{y}$: $V = \pi \int_0^4 x^2\,dy = \pi \int_0^4 y\,dy = 8\pi$. $\frac{16\pi}{3}$ uses $\sqrt{y}$ without squaring; 16π is the enclosing cylinder; $\frac{128\pi}{5}$ rotates the region under the curve about the x-axis.

12. C. $\log_e\left(2+\sqrt{3}\right)$

$\frac{dy}{dx} = -\tan(x)$, so $\sqrt{1+\tan^2(x)} = \sec(x)$ on this interval and

$L = \int_0^{\frac{\pi}{3}} \sec(x)\,dx = \log_e\left(\sec\frac{\pi}{3} + \tan\frac{\pi}{3}\right) = \log_e(2+\sqrt{3}) \approx 1.317$ (by CAS). $\sqrt{3}$ integrates $\sec^2(x)$ instead; $\log_e(2)$ is only the vertical drop.

13. A. $\int_0^1 \left(u^{\frac{1}{2}} - u^{\frac{3}{2}}\right)du$

$x = 1-u$ and $dx = -du$; the terminals $x = 0, 1$ become $u = 1, 0$. So the integral is

$\int_1^0 (1-u)\sqrt{u}\,(-du) = \int_0^1 \left(u^{\frac{1}{2}} - u^{\frac{3}{2}}\right)du$. B forgets the factor -1 from $dx = -du$; C reverses the terminals as well; D drops the square root.

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PREVIEW

That’s the end of the preview answers

These are the answers to the 13 preview questions. The full answer key covers all 26 questions in Specialist Mathematics Unit 3 & 4 — Examination 2 (Practice 1), with the working behind every mark.

STILL TO COME	QUESTIONS	MARKS
Section A — multiple choice	7	7
Section B — extended response	6	60

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