Specialist Mathematics Unit 3 & 4 — Examination 1 (Practice 2) — Answers & Explanations

Answers to the 5 questions in the free preview.

Examination 1 — technology-free

1. (3 marks in total)

True for $n = 1$ (both sides equal 2); if true for $n = k$, adding $(k+1)2^{k+1}$ gives $k\,2^{k+2} + 2$, the formula for $n = k+1$.

[3 marks]

Base case: the left side is $1 \times 2 = 2$ and the right side is $0 \times 4 + 2 = 2$. Inductive step: assume $\sum_{r=1}^{k} r\,2^r = (k-1)2^{k+1} + 2$. Then

$$\sum_{r=1}^{k+1} r\,2^r = (k-1)2^{k+1} + 2 + (k+1)2^{k+1} = 2k \times 2^{k+1} + 2 = k\,2^{k+2} + 2 = \big((k+1)-1\big)2^{(k+1)+1} + 2,$$

which is the statement for $n = k+1$. By the principle of mathematical induction the statement is true for all $n \in \mathbb{N}$. Marks: 1 base case; 1 adding the $(k+1)$th term to the assumption; 1 simplifying to the required form and concluding.

2. (4 marks in total)

a. $P(2) = 8 + 2 - 10 = 0$

[1 mark]

Substitute $z = 2$. (By the factor theorem, $z - 2$ is then a factor of $P(z)$.)

b. $z = -1 + 2i$ and $z = -1 - 2i$

[2 marks]

$P(z) = (z-2)(z^2 + 2z + 5)$, found by division or by matching coefficients. $z^2 + 2z + 5 = (z+1)^2 + 4 = 0$ gives $z = -1 \pm 2i$. 1 mark for the quadratic factor, 1 mark for the solutions.

c. 6 square units

[1 mark]

The vertices are $(2, 0)$, $(-1, 2)$ and $(-1, -2)$. The side from $(-1, 2)$ to $(-1, -2)$ has length 4 and the perpendicular distance from $(2, 0)$ to it is 3, so the area is $\frac{1}{2} \times 4 \times 3 = 6$.

3. (5 marks in total)

a. Asymptotes $x = 0$, $x = \frac{\pi}{2}$, $x = \pi$; local minimum $\left(\frac{\pi}{4}, 2\right)$; local maximum $\left(\frac{3\pi}{4}, 0\right)$.

[2 marks]

$\operatorname{cosec}(2x) = \frac{1}{\sin(2x)}$ is undefined where $\sin(2x) = 0$: $x = 0, \frac{\pi}{2}, \pi$. On $\left(0, \frac{\pi}{2}\right)$ the graph is a U-shape with minimum $1 + 1 = 2$ at $x = \frac{\pi}{4}$; on $\left(\frac{\pi}{2}, \pi\right)$ it is an inverted U with maximum $-1 + 1 = 0$ at $x = \frac{3\pi}{4}$, touching the x-axis there. 1 mark for the asymptotes, 1 mark for the shape with the turning points.

b. $x = \dfrac{\pi}{12}$ or $x = \dfrac{5\pi}{12}$

[2 marks]

$\operatorname{cosec}(2x) = 2$, so $\sin(2x) = \frac{1}{2}$ with $2x \in (0, 2\pi)$: $2x = \frac{\pi}{6}$ or $\frac{5\pi}{6}$. 1 mark for $\sin(2x) = \frac{1}{2}$, 1 mark for both solutions. Common error: giving only $\frac{\pi}{12}$.

c. $(-\infty, 0] \cup [2, \infty)$

[1 mark]

$\operatorname{cosec}(2x)$ takes every value in $(-\infty, -1] \cup [1, \infty)$ on this domain; add 1.

4. (4 marks in total)

a. $(0, 0)$ and $(2, 4)$

[1 mark]

$x^2 = 2x$ gives $x = 0$ or $x = 2$.

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b. $\dfrac{8\pi}{3}$ cubic units

[3 marks]

About the y-axis, for $0 \le y \le 4$ the outer radius is $x = \sqrt{y}$ (from $y = x^2$) and the inner radius is $x = \frac{y}{2}$ (from $y = 2x$).

$$V = \pi \int_0^4 \left(y - \frac{y^2}{4} \right) dy = \pi \left[\frac{y^2}{2} - \frac{y^3}{12} \right]_0^4 = \pi \left(8 - \frac{16}{3} \right) = \frac{8\pi}{3}.$$

Marks: 1 for expressing the radii in terms of y; 1 for the integral; 1 for the value. Common error: using the x-axis formula $\pi \int y^2 \, dx$.

5. (3 marks in total)

$v = 2\sqrt{3}$ m s^{-1} when $x = 1$

[3 marks]

Use $a = \dfrac{d}{dx}\left(\frac{1}{2}v^2\right)$: $\frac{1}{2}v^2 = x^2 + 3x + c$, and $v = 2$ at $x = 0$ gives $c = 2$. So $v^2 = 2x^2 + 6x + 4$, and at $x = 1$, $v^2 = 12$; $v > 0$, so $v = 2\sqrt{3}$. Marks: 1 for the antiderivative of $\frac{1}{2}v^2$; 1 for the constant; 1 for the value.

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PREVIEW

That’s the end of the preview answers

These are the answers to the 5 preview questions. The full answer key covers all 10 questions in Specialist Mathematics Unit 3 & 4 — Examination 1 (Practice 2), with the working behind every mark.

STILL TO COME	QUESTIONS	MARKS
Examination 1 — technology-free	5	21

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