Specialist Mathematics Unit 3 & 4 — Examination 2 (Practice 2) — Answers & Explanations

Answers to the 13 questions in the free preview.

Section A — multiple choice

1. A. If the diagonals of a quadrilateral are equal in length, then it is a square.

The converse of "if P then Q" is "if Q then P". B is the inverse, C is the contrapositive (equivalent to the original) and D is the negation. The converse here is false: a rectangle has equal diagonals.

2. C. $\sqrt{2}+\sqrt{3}$ is rational

A proof by contradiction assumes the negation of the statement to be proved and derives a contradiction. The negation of "$\sqrt{2}+\sqrt{3}$ is irrational" is "$\sqrt{2}+\sqrt{3}$ is rational".

3. D. $b=\pm 4$

A single vertical asymptote needs the denominator to have exactly one (repeated) root: $b^2-16=0$, so $b=\pm 4$. The root is $x=\mp 2$, which is not a zero of the numerator. For $-4<b<4$ there are no vertical asymptotes.

4. B. $x=\dfrac{3\pi}{4}$ and $x=\dfrac{7\pi}{4}$

$\sec(u)$ is undefined where $\cos(u)=0$: $x-\frac{\pi}{4}=\frac{\pi}{2}$ or $\frac{3\pi}{2}$, so $x=\frac{3\pi}{4}$ or $\frac{7\pi}{4}$. C ignores the translation; A gives the zeros of $\sin\left(x-\frac{\pi}{4}\right)$.

5. A. $2\operatorname{cis}\left(\dfrac{\pi}{6}\right)$

$-8+8\sqrt{3}i=16\operatorname{cis}\left(\frac{2\pi}{3}\right)$, so $z=16^{\frac{1}{4}}\operatorname{cis}\left(\frac{2\pi}{3}\times\frac{1}{4}+\frac{k\pi}{2}\right)=2\operatorname{cis}\left(\frac{\pi}{6}+\frac{k\pi}{2}\right)$. With $k=0$: $2\operatorname{cis}\left(\frac{\pi}{6}\right)$. Check: $2^4\operatorname{cis}\left(\frac{4\pi}{6}\right)=16\operatorname{cis}\left(\frac{2\pi}{3}\right)$. C forgets the fourth root of the modulus.

6. C. the circle with centre -1 and radius 2

With $z=x+iy$: $(x-3)^2+y^2=4(x^2+y^2)$, so $3x^2+3y^2+6x-9=0$, $x^2+2x+y^2=3$ and $(x+1)^2+y^2=4$: centre -1, radius 2. ($|z-a|=|z-b|$ gives a line; a ratio other than 1 gives a circle.)

7. B. a real number

With $z=r\operatorname{cis}(\theta)$, $\frac{z}{\bar{z}}=\operatorname{cis}(2\theta)$ and $\frac{\bar{z}}{z}=\operatorname{cis}(-2\theta)$, so the sum is $2\cos(2\theta)$, which is real. It equals 2 only when z is real, and 0 when $\theta=\pm\frac{\pi}{4}$ or $\pm\frac{3\pi}{4}$.

8. D. 5.17

$y_1=3+0.5\sqrt{1+3}=4$ at $x=1.5$; $y_2=4+0.5\sqrt{1.5+4}\approx 5.17$ at $x=2$. 4.00 stops after one step; 5.00 reuses the first gradient; 5.12 forgets to advance x in the second step.

9. C. 6.49

The rate $0.3P\left(1-\frac{P}{800}\right)$ is a quadratic in P, greatest at $P=400$. The solution is $P=\dfrac{800}{1+7e^{-0.3t}}$, and $P=400$ when $7e^{-0.3t}=1$: $t=\frac{\log_e(7)}{0.3}\approx 6.49$. 400 is the population at that time, not the time; 3.24 uses 0.6 in place of 0.3.

10. B. $\dfrac{2}{\pi}$

By similar triangles $r=\frac{h}{2}$, so $V=\frac{1}{3}\pi\left(\frac{h}{2}\right)^2h=\frac{\pi h^3}{12}$ and $\frac{dV}{dh}=\frac{\pi h^2}{4}$. Then $\frac{dh}{dt}=\frac{dV}{dt}\div\frac{dV}{dh}=0.5\div\frac{\pi}{4}=\frac{2}{\pi}$ at $h=1$. D inverts the chain rule.

11. A. $e-2$

By parts twice: $\int x^2e^x\,dx=x^2e^x-2\int xe^x\,dx=(x^2-2x+2)e^x+c$. Evaluating: $e(1-2+2)-2=e-2$.

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12. D. $\pi \int_0^4 \sqrt{4x+1}\,dx$

$S = 2\pi \int_0^4 y\sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx$ with $\frac{dy}{dx} = \frac{1}{2\sqrt{x}}$: $2\pi\sqrt{x}\sqrt{1+\frac{1}{4x}} = 2\pi\sqrt{x+\frac{1}{4}} = \pi\sqrt{4x+1}$. B is the volume of the solid, not its surface area.

13. B. 4

$v^2 = u^2 + 2as$: $0 = 625 + 100a$, so $a = -6.25$. Then $t = \frac{v-u}{a} = \frac{-25}{-6.25} = 4$. (Or $s = \frac{1}{2}(u+v)t$: $50 = 12.5t$.) 6.25 is the deceleration, not the time.

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PREVIEW

That’s the end of the preview answers

These are the answers to the 13 preview questions. The full answer key covers all 26 questions in Specialist Mathematics Unit 3 & 4 — Examination 2 (Practice 2), with the working behind every mark.

STILL TO COME	QUESTIONS	MARKS
Section A — multiple choice	7	7
Section B — extended response	6	60

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