EXAM PAPER

Specialist Mathematics Unit 3 & 4 — Examination 2 (Practice 2)

PREVIEW
This file has the first 13 of the 26 questions in this paper. The rest is locked: it comes with a PrepNest plan, or with the paper itself for VCE.

Reading time: 15 minutes (no writing)
Writing time: 120 minutes
Total time: 135 minutes

SECTIONS IN THIS PAPER

- Section A — multiple choice — 45 min (calculator allowed) — 13 questions, 13 marks
- Section B — extended response — 75 min (calculator allowed) — 0 questions, 0 marks

A formula sheet is printed at the end of this paper.

INSTRUCTIONS

You are not permitted to write during reading time — you may only read the paper and plan your approach. Answer every question you can. Write your working in the space provided for long-answer questions. Marking guidance and full explanations are provided in the separate answer key.

PrepNest

Section A — multiple choice

45 minutes • Calculator allowed

INSTRUCTIONS

- Answer all questions. Choose the response that is correct for the question.
- A correct answer scores 1; an incorrect answer scores 0. Marks will not be deducted for incorrect answers.
- Unless otherwise indicated, the diagrams in this book are not drawn to scale.
- Take the acceleration due to gravity to have magnitude g m s^{-2}, where g = 9.8.

Question 1

The converse of the statement ‘If a quadrilateral is a square, then its diagonals are equal in length’ is

A. If the diagonals of a quadrilateral are equal in length, then it is a square.

B. If a quadrilateral is not a square, then its diagonals are not equal in length.

C. If the diagonals of a quadrilateral are not equal in length, then it is not a square.

D. A quadrilateral is a square and its diagonals are not equal in length.

Question 2

A proof by contradiction that $\sqrt{2}+\sqrt{3}$ is irrational would begin by assuming that

A. $\sqrt{2}$ and $\sqrt{3}$ are both rational

B. $\sqrt{2}+\sqrt{3}$ is irrational

C. $\sqrt{2}+\sqrt{3}$ is rational

D. $\left(\sqrt{2}+\sqrt{3}\right)^2$ is rational

Question 3

The graph of $y=\dfrac{x}{x^2+bx+4}$, where $b\in\mathbb{R}$, has exactly one vertical asymptote when

A. $b=0$

B. $b=\pm 2$

C. $-4<b<4$

D. $b=\pm 4$

PrepNest

Question 4

For $0 \leq x \leq 2\pi$, the graph of $y = \sec\left(x - \frac{\pi}{4}\right)$ has vertical asymptotes with equations

A. $x = \frac{\pi}{4}$ and $x = \frac{5\pi}{4}$

B. $x = \frac{3\pi}{4}$ and $x = \frac{7\pi}{4}$

C. $x = \frac{\pi}{2}$ and $x = \frac{3\pi}{2}$

D. $x = \frac{\pi}{4}$ and $x = \frac{3\pi}{4}$

Question 5

One of the solutions of $z^4 = -8 + 8\sqrt{3}i$ is

A. $2\,\mathrm{cis}\left(\frac{\pi}{6}\right)$

B. $2\,\mathrm{cis}\left(\frac{\pi}{3}\right)$

C. $4\,\mathrm{cis}\left(\frac{\pi}{6}\right)$

D. $2\,\mathrm{cis}\left(\frac{\pi}{12}\right)$

Question 6

In the complex plane, the set of points satisfying $|z - 3| = 2|z|$ is

A. the circle with centre 1 and radius 2

B. the line $\mathrm{Re}(z) = 1$

C. the circle with centre -1 and radius 2

D. the circle with centre -1 and radius 4

Question 7

For any non-zero complex number z, the expression $\frac{z}{\overline{z}} + \frac{\overline{z}}{z}$ is always

A. equal to 2

B. a real number

C. purely imaginary

D. equal to 0

PrepNest

Question 8

Euler's method with a step size of 0.5 is used to approximate the solution of $\frac{dy}{dx} = \sqrt{x+y}$, where $y(1) = 3$. The approximation to $y(2)$, correct to two decimal places, is

A. 4.00
B. 5.00
C. 5.12
D. 5.17

Question 9

A population P is modelled by $\frac{dP}{dt} = 0.3P\left(1 - \frac{P}{800}\right)$, where $P(0) = 100$ and t is in years. The population is increasing most rapidly when t, correct to two decimal places, equals

A. 3.24
B. 4.00
C. 6.49
D. 400.00

Question 10

Water is poured at 0.5 m³ per minute into a tank in the shape of an inverted right circular cone with radius 2 m and height 4 m. When the depth of water is 1 m, the depth is increasing, in metres per minute, at the rate

A. $\frac{1}{8\pi}$
B. $\frac{2}{\pi}$
C. $\frac{1}{2\pi}$
D. $\frac{8}{\pi}$

Question 11

$\int_0^1 x^2 e^x \, dx$ is equal to

A. $e - 2$
B. 1
C. $e - 1$
D. $2e - 2$

PrepNest

Question 12

The curve $y = \sqrt{x}$, $0 \leq x \leq 4$, is rotated about the x-axis. The area of the curved surface generated is given by

A. $2\pi \int_0^4 \sqrt{4x+1}\, dx$

B. $\pi \int_0^4 x\, dx$

C. $\pi \int_0^4 \sqrt{x+1}\, dx$

D. $\pi \int_0^4 \sqrt{4x+1}\, dx$

Question 13

A car travelling at 25 m s^{-1} brakes with constant deceleration and comes to rest after travelling 5 0 m. The time taken to stop, in seconds, is

A. 2

B. 4

C. 6.25

D. 8

PrepNest

FORMULA SHEET

Specialist Mathematics formulas

Mensuration	
area of a circle segment	$\frac{r^2}{2}(\theta - \sin(\theta))$
volume of a cylinder	$\pi r^2 h$
volume of a cone	$\frac{1}{3}\pi r^2 h$
volume of a pyramid	$\frac{1}{3}Ah$
volume of a sphere	$\frac{4}{3}\pi r^3$
area of a triangle	$\frac{1}{2}bc\sin(A)$
sine rule	$\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}$
cosine rule	$c^2 = a^2 + b^2 - 2ab\cos(C)$

Algebra, number and structure (complex numbers)	
$z = x + iy = r(\cos(\theta) + i\sin(\theta)) = r\,\mathrm{cis}(\theta)$	$\lvert z\rvert = \sqrt{x^2 + y^2} = r$
$-\pi < \mathrm{Arg}(z) \leq \pi$	$z_1 z_2 = r_1 r_2\,\mathrm{cis}(\theta_1 + \theta_2)$
$\frac{z_1}{z_2} = \frac{r_1}{r_2}\mathrm{cis}(\theta_1 - \theta_2)$	de Moivre's theorem $z^n = r^n\,\mathrm{cis}(n\theta)$

Data analysis, probability and statistics		
for independent random variables $X_1, X_2, \ldots, X_n$	$\mathrm{E}(aX_1 + b) = a\,\mathrm{E}(X_1) + b$	$\mathrm{Var}(aX_1 + b) = a^2\,\mathrm{Var}(X_1)$
	$\mathrm{E}(a_1X_1 + a_2X_2 + \cdots + a_nX_n) = a_1\,\mathrm{E}(X_1) + a_2\,\mathrm{E}(X_2) + \cdots + a_n\,\mathrm{E}(X_n)$	
	$\mathrm{Var}(a_1X_1 + a_2X_2 + \cdots + a_nX_n) = a_1^2\,\mathrm{Var}(X_1) + a_2^2\,\mathrm{Var}(X_2) + \cdots + a_n^2\,\mathrm{Var}(X_n)$	
for independent identically distributed variables $X_1, X_2, \ldots, X_n$	$\mathrm{E}(X_1 + X_2 + \cdots + X_n) = n\mu$	$\mathrm{Var}(X_1 + X_2 + \cdots + X_n) = n\sigma^2$
approximate confidence interval for μ	$\left(\bar{x} - z\frac{s}{\sqrt{n}},\ \bar{x} + z\frac{s}{\sqrt{n}}\right)$	
distribution of sample mean $\bar{X}$	mean $\mathrm{E}(\bar{X}) = \mu$	variance $\mathrm{Var}(\bar{X}) = \frac{\sigma^2}{n}$

PrepNest

Circular (trigonometric) functions	
$\cos^2(x)+\sin^2(x)=1$	
$1+\tan^2(x)=\sec^2(x)$	$\cot^2(x)+1=\operatorname{cosec}^2(x)$
$\sin(x+y)=\sin(x)\cos(y)+\cos(x)\sin(y)$	$\sin(x-y)=\sin(x)\cos(y)-\cos(x)\sin(y)$
$\cos(x+y)=\cos(x)\cos(y)-\sin(x)\sin(y)$	$\cos(x-y)=\cos(x)\cos(y)+\sin(x)\sin(y)$
$\tan(x+y)=\dfrac{\tan(x)+\tan(y)}{1-\tan(x)\tan(y)}$	$\tan(x-y)=\dfrac{\tan(x)-\tan(y)}{1+\tan(x)\tan(y)}$
$\cos(2x)=\cos^2(x)-\sin^2(x)=2\cos^2(x)-1=1-2\sin^2(x)$	
$\sin(2x)=2\sin(x)\cos(x)$	$\tan(2x)=\dfrac{2\tan(x)}{1-\tan^2(x)}$
$\sin^2(ax)=\dfrac{1}{2}(1-\cos(2ax))$	$\cos^2(ax)=\dfrac{1}{2}(1+\cos(2ax))$

Calculus	
$\dfrac{d}{dx}(x^n)=nx^{n-1}$	$\displaystyle\int x^n\,dx=\frac{1}{n+1}x^{n+1}+c,\ n\neq -1$
$\dfrac{d}{dx}(e^{ax})=ae^{ax}$	$\displaystyle\int e^{ax}\,dx=\frac{1}{a}e^{ax}+c$
$\dfrac{d}{dx}(\log_e(x))=\dfrac{1}{x}$	$\displaystyle\int \frac{1}{x}\,dx=\log_e\lvert x\rvert+c$
$\dfrac{d}{dx}(\sin(ax))=a\cos(ax)$	$\displaystyle\int \sin(ax)\,dx=-\frac{1}{a}\cos(ax)+c$
$\dfrac{d}{dx}(\cos(ax))=-a\sin(ax)$	$\displaystyle\int \cos(ax)\,dx=\frac{1}{a}\sin(ax)+c$
$\dfrac{d}{dx}(\tan(ax))=a\sec^2(ax)$	$\displaystyle\int \sec^2(ax)\,dx=\frac{1}{a}\tan(ax)+c$
$\dfrac{d}{dx}(\cot(ax))=-a\operatorname{cosec}^2(ax)$	$\displaystyle\int \operatorname{cosec}^2(ax)\,dx=-\frac{1}{a}\cot(ax)+c$
$\dfrac{d}{dx}(\sec(ax))=a\sec(ax)\tan(ax)$	$\displaystyle\int \sec(ax)\tan(ax)\,dx=\frac{1}{a}\sec(ax)+c$
$\dfrac{d}{dx}(\operatorname{cosec}(ax))=-a\operatorname{cosec}(ax)\cot(ax)$	$\displaystyle\int \operatorname{cosec}(ax)\cot(ax)\,dx=-\frac{1}{a}\operatorname{cosec}(ax)+c$
$\dfrac{d}{dx}\left(\sin^{-1}(ax)\right)=\dfrac{a}{\sqrt{1-(ax)^2}}$	$\displaystyle\int \frac{1}{\sqrt{a^2-x^2}}\,dx=\sin^{-1}\left(\frac{x}{a}\right)+c,\ a>0$
$\dfrac{d}{dx}\left(\cos^{-1}(ax)\right)=\dfrac{-a}{\sqrt{1-(ax)^2}}$	$\displaystyle\int \frac{-1}{\sqrt{a^2-x^2}}\,dx=\cos^{-1}\left(\frac{x}{a}\right)+c,\ a>0$
$\dfrac{d}{dx}\left(\tan^{-1}(ax)\right)=\dfrac{a}{1+(ax)^2}$	$\displaystyle\int \frac{a}{a^2+x^2}\,dx=\tan^{-1}\left(\frac{x}{a}\right)+c$
$\displaystyle\int (ax+b)^{-1}\,dx=\frac{1}{a}\log_e\lvert ax+b\rvert+c$	$\displaystyle\int (ax+b)^n\,dx=\frac{1}{a(n+1)}(ax+b)^{n+1}+c,\ n\neq -1$

PrepNest

Calculus — continued	
product rule	$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$
quotient rule	$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$
chain rule	$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$
integration by parts	$\int u\frac{dv}{dx}\,dx = uv - \int v\frac{du}{dx}\,dx$
Euler's method	If $\frac{dy}{dx} = f(x, y)$, $x_0 = a$ and $y_0 = b$, then $x_{n+1} = x_n + h$ and $y_{n+1} = y_n + hf(x_n, y_n)$.
arc length parametric	$\int_{t_1}^{t_2} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt$
surface area Cartesian about the x-axis	$\int_{x_1}^{x_2} 2\pi y\sqrt{1 + \left(\frac{dy}{dx}\right)^2}\,dx$
surface area Cartesian about the y-axis	$\int_{y_1}^{y_2} 2\pi x\sqrt{1 + \left(\frac{dx}{dy}\right)^2}\,dy$
surface area parametric about the x-axis	$\int_{t_1}^{t_2} 2\pi y\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt$
surface area parametric about the y-axis	$\int_{t_1}^{t_2} 2\pi x\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt$

Kinematics		
acceleration	$a = \frac{d^2x}{dt^2} = \frac{dv}{dt} = v\frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$	
constant acceleration formulas	$v = u + at$	$s = ut + \frac{1}{2}at^2$
	$v^2 = u^2 + 2as$	$s = \frac{1}{2}(u + v)t$

PrepNest

Vectors in two and three dimensions		
	$\underset{\sim}{r}(t) = x(t)\underset{\sim}{i} + y(t)\underset{\sim}{j} + z(t)\underset{\sim}{k}$	$\lvert\underset{\sim}{r}(t)\rvert = \sqrt{x(t)^2 + y(t)^2 + z(t)^2}$
	$\dot{\underset{\sim}{r}}(t) = \frac{d\underset{\sim}{r}}{dt} = \frac{dx}{dt}\underset{\sim}{i} + \frac{dy}{dt}\underset{\sim}{j} + \frac{dz}{dt}\underset{\sim}{k}$	
for $\underset{\sim}{r}_1 = x_1\underset{\sim}{i} + y_1\underset{\sim}{j} + z_1\underset{\sim}{k}$ and $\underset{\sim}{r}_2 = x_2\underset{\sim}{i} + y_2\underset{\sim}{j} + z_2\underset{\sim}{k}$	vector scalar product $\underset{\sim}{r}_1 \cdot \underset{\sim}{r}_2 = \lvert\underset{\sim}{r}_1\rvert\lvert\underset{\sim}{r}_2\rvert\cos(\theta) = x_1x_2 + y_1y_2 + z_1z_2$	
	vector cross product $\underset{\sim}{r}_1 \times \underset{\sim}{r}_2 = \begin{vmatrix} \underset{\sim}{i} & \underset{\sim}{j} & \underset{\sim}{k} \\ x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \end{vmatrix} = (y_1z_2 - y_2z_1)\underset{\sim}{i} + (x_2z_1 - x_1z_2)\underset{\sim}{j} + (x_1y_2 - x_2y_1)\underset{\sim}{k}$	
vector equation of a line	$\underset{\sim}{r}(t) = \underset{\sim}{r}_1 + t\underset{\sim}{r}_2 = (x_1 + x_2t)\underset{\sim}{i} + (y_1 + y_2t)\underset{\sim}{j} + (z_1 + z_2t)\underset{\sim}{k}$	
parametric equation of a line	$x(t) = x_1 + x_2t \quad y(t) = y_1 + y_2t \quad z(t) = z_1 + z_2t$	
vector equation of a plane	$\underset{\sim}{r}(s,t) = \underset{\sim}{r}_0 + s\underset{\sim}{r}_1 + t\underset{\sim}{r}_2 = (x_0 + x_1s + x_2t)\underset{\sim}{i} + (y_0 + y_1s + y_2t)\underset{\sim}{j} + (z_0 + z_1s + z_2t)\underset{\sim}{k}$	
parametric equation of a plane	$x(s,t) = x_0 + x_1s + x_2t \quad y(s,t) = y_0 + y_1s + y_2t \quad z(s,t) = z_0 + z_1s + z_2t$	
Cartesian equation of a plane	$ax + by + cz = d$	

END OF FORMULA SHEET

PrepNest

LOCKED The rest of the paper comes with a PrepNest plan · prepnest.com.au/pricing

14.

15.

16.

17.

18.

PrepNest

LOCKED The rest of the paper comes with a PrepNest plan · prepnest.com.au/pricing

19.

20.

1.

LOCKED The rest of the paper comes with a PrepNest plan · prepnest.com.au/pricing

2.

3.

4.

PrepNest

LOCKED The rest of the paper comes with a PrepNest plan · prepnest.com.au/pricing

5.

6.

PrepNest

13 questions locked, up to question 6.

PREVIEW

That’s the end of the preview

You’ve seen the first 13 of the 26 questions in Specialist Mathematics Unit 3 & 4 — Examination 2 (Practice 2). The full paper is worth 80 marks.

STILL TO COME	QUESTIONS	MARKS
Section A — multiple choice	7	7
Section B — extended response	6	60

The full paper comes with

- Every question, printed exactly like the ones in this preview
- The complete answer key, with the working behind every mark
- A topic report after marking, showing what to practice next

Get the full paper at
prepnest.com.au/practice/exams/specialist_maths-year_12-2-exam2

PrepNest