Specialist Mathematics Unit 3 & 4 — Examination 1 (Practice 3) — Answers & Explanations

Answers to the 5 questions in the free preview.

Examination 1 — technology-free

1. (4 marks in total)

a. $3! = 6 < 8 = 2^3$; $4! = 24 > 16 = 2^4$

[1 mark]

Direct evaluation of both sides.

b. Base case $n = 4$ from part a; if $k! > 2^k$ with $k \geq 4$, then $(k+1)! = (k+1)k! > (k+1)2^k > 2 \times 2^k = 2^{k+1}$.

[3 marks]

Assume $k! > 2^k$ for some integer $k \geq 4$. Then $(k+1)! = (k+1) \times k! > (k+1) \times 2^k$, and since $k + 1 > 2$, $(k+1)2^k > 2^{k+1}$. So $(k+1)! > 2^{k+1}$, and the statement is true for $n = k + 1$. With the base case $n = 4$, it is true for all $n \geq 4$ by induction. Marks: 1 for using the assumption; 1 for justifying $k + 1 > 2$; 1 for the conclusion referring to $n \geq 4$.

2. (4 marks in total)

a. $|0 - (1 + i)| = \sqrt{1 + 1} = \sqrt{2}$

[1 mark]

The origin is at distance $\sqrt{2}$ from the centre $1 + i$, which equals the radius.

b. The closed disc with centre $(1, 1)$ and radius $\sqrt{2}$, boundary included; it passes through 0, 2 and $2i$.

[1 mark]

The circle also passes through 2 and $2i$, since $|2 - (1 + i)| = |1 - i| = \sqrt{2}$. Shade the inside and draw the boundary as a solid curve.

c. $2\sqrt{2}$

[1 mark]

The furthest point from the origin is diametrically opposite 0, at $2 + 2i$: $|1 + i| + \sqrt{2} = 2\sqrt{2}$.

d. $-\dfrac{\pi}{4} < \operatorname{Arg}(z) < \dfrac{3\pi}{4}$

[1 mark]

At the origin the circle's tangent is perpendicular to the radius, which has direction $\frac{\pi}{4}$; so the tangent has directions $-\frac{\pi}{4}$ and $\frac{3\pi}{4}$. Points of S near 0 approach these arguments but never reach them.

3. (4 marks in total)

a. Domain $[0, 2]$, range $[-\pi, \pi]$

[2 marks]

$-1 \leq 1 - x \leq 1$ gives $0 \leq x \leq 2$. $\sin^{-1}$ has range $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$, doubled. 1 mark each.

b. $f'(x) = -\dfrac{2}{\sqrt{2x - x^2}}$

[1 mark]

$f'(x) = 2 \times \dfrac{-1}{\sqrt{1 - (1 - x)^2}}$, and $1 - (1 - x)^2 = 2x - x^2$.

c. $-\dfrac{4}{\sqrt{3}} = -\dfrac{4\sqrt{3}}{3}$

[1 mark]

$2x - x^2 = 1 - \frac{1}{4} = \frac{3}{4}$, so $f'\left(\frac{1}{2}\right) = -\dfrac{2}{\sqrt{3}/2} = -\dfrac{4}{\sqrt{3}}$.

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4. (3 marks in total)

$$\frac{56\pi}{3}$$

[3 marks]

$\frac{dy}{dx} = \frac{1}{\sqrt{x}}$, so $2\pi y\sqrt{1+\left(\frac{dy}{dx}\right)^2} = 4\pi\sqrt{x}\sqrt{\frac{x+1}{x}} = 4\pi\sqrt{x+1}$.

$$S = 4\pi\int_0^3 \sqrt{x+1}\,dx = 4\pi \times \frac{2}{3}\left[(x+1)^{\frac{3}{2}}\right]_0^3 = \frac{8\pi}{3}(8-1) = \frac{56\pi}{3}.$$

Marks: 1 for the surface area integral; 1 for simplifying the integrand; 1 for the value.

5. (4 marks in total)

a. $\cos^3(x) = \cos^2(x)\cos(x) = (1-\sin^2(x))\cos(x)$

[1 mark]

Use $\cos^2(x) = 1-\sin^2(x)$ and expand.

b. $$\frac{2}{15}$$

[3 marks]

Let $u = \sin(x)$, so $\frac{du}{dx} = \cos(x)$; the terminals become $u = 0$ and $u = 1$.

$$\int_0^1 (u^2 - u^4)\,du = \frac{1}{3} - \frac{1}{5} = \frac{2}{15}.$$

Marks: 1 for the substitution; 1 for the new terminals and integrand; 1 for the value.

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PREVIEW

That’s the end of the preview answers

These are the answers to the 5 preview questions. The full answer key covers all 10 questions in Specialist Mathematics Unit 3 & 4 — Examination 1 (Practice 3), with the working behind every mark.

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- Every question, printed exactly like the ones in this preview
- The complete answer key, with the working behind every mark
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