EXAM PAPER

Specialist Mathematics Unit 3 & 4 — Examination 1 (Practice 3)

PREVIEW
This file has the first 5 of the 10 questions in this paper. The rest is locked: it comes with a PrepNest plan, or with the paper itself for VCE.

Reading time: 15 minutes (no writing)
Writing time: 60 minutes
Total time: 75 minutes

SECTIONS IN THIS PAPER

- Examination 1 — technology-free — 60 min (no calculator) — 5 questions, 19 marks

A formula sheet is printed at the end of this paper.

INSTRUCTIONS

You are not permitted to write during reading time — you may only read the paper and plan your approach. Answer every question you can. Write your working in the space provided for long-answer questions. Marking guidance and full explanations are provided in the separate answer key.

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YEAR 12 SPECIALIST MATHEMATICS (NON-CALCULATOR)

Examination 1 — technology-free

60 minutes • No calculator

INSTRUCTIONS

- Answer all questions in the spaces provided.
- In all questions where a numerical answer is required, an exact value must be given, unless otherwise specified.
- In questions where more than one mark is available, appropriate working must be shown.
- Unless otherwise indicated, the diagrams in this book are not drawn to scale.

Question 1

(4 marks)

Consider the statement $n! > 2^n$, where $n \in \mathbb{N}$.

a. Show that the statement is false for $n = 3$ and true for $n = 4$. 1 mark

b. Prove by mathematical induction that $n! > 2^n$ for all integers $n \geq 4$. 3 marks

Question 2

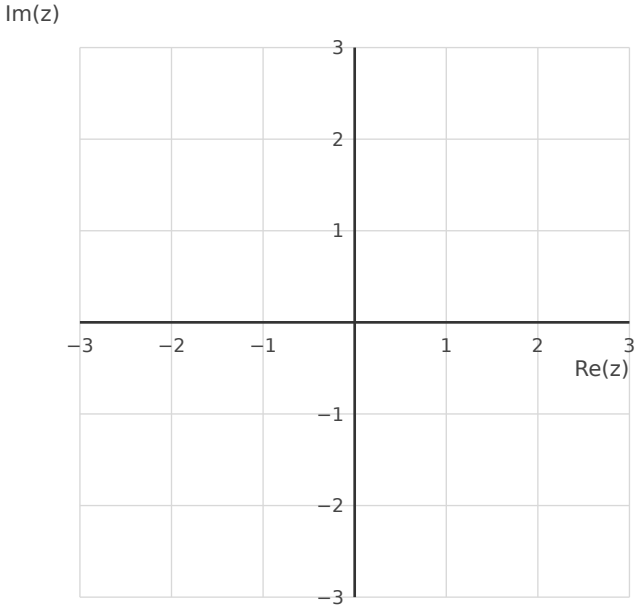
(4 marks)

Let $S = \left\{ z \in \mathbb{C} : |z - (1 + i)| \leq \sqrt{2} \right\}$.

a. Show that the boundary of S passes through the origin. 1 mark

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b. Sketch S on the Argand diagram below. 1 mark



c. Find the maximum value of $|z|$ for $z \in S$. 1 mark

d. Find the set of possible values of $\text{Arg}(z)$ for $z \in S$, $z \neq 0$. 1 mark

Question 3

(4 marks)

Let $f(x) = 2\sin^{-1}(1-x)$.

a. State the maximal domain and the range of f. 2 marks

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b. Find $f'(x)$, expressing your answer in terms of x with no inverse circular functions. 1 mark

c. Find the gradient of the graph of f at $x = \frac{1}{2}$. 1 mark

Question 4

(3 marks)

The curve $y = 2\sqrt{x}$, $0 \leq x \leq 3$, is rotated about the x-axis. Find the area of the curved surface generated.

Question 5

(4 marks)

Consider $\int_0^{\frac{\pi}{2}} \sin^2(x)\cos^3(x)\,dx$.

a. Show that $\sin^2(x)\cos^3(x) = \left(\sin^2(x) - \sin^4(x)\right)\cos(x)$. 1 mark

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b. Hence, evaluate the integral. 3 marks

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FORMULA SHEET

Specialist Mathematics formulas

Mensuration	
area of a circle segment	$\frac{r^2}{2}(\theta - \sin(\theta))$
volume of a cylinder	$\pi r^2 h$
volume of a cone	$\frac{1}{3}\pi r^2 h$
volume of a pyramid	$\frac{1}{3}Ah$
volume of a sphere	$\frac{4}{3}\pi r^3$
area of a triangle	$\frac{1}{2}bc\sin(A)$
sine rule	$\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}$
cosine rule	$c^2 = a^2 + b^2 - 2ab\cos(C)$

Algebra, number and structure (complex numbers)	
$z = x + iy = r(\cos(\theta) + i\sin(\theta)) = r\,\mathrm{cis}(\theta)$	$\lvert z\rvert = \sqrt{x^2 + y^2} = r$
$-\pi < \mathrm{Arg}(z) \leq \pi$	$z_1 z_2 = r_1 r_2\,\mathrm{cis}(\theta_1 + \theta_2)$
$\frac{z_1}{z_2} = \frac{r_1}{r_2}\mathrm{cis}(\theta_1 - \theta_2)$	de Moivre's theorem $z^n = r^n\,\mathrm{cis}(n\theta)$

Data analysis, probability and statistics		
for independent random variables $X_1, X_2, \ldots, X_n$	$\mathrm{E}(aX_1 + b) = a\,\mathrm{E}(X_1) + b$	$\mathrm{Var}(aX_1 + b) = a^2\,\mathrm{Var}(X_1)$
	$\mathrm{E}(a_1X_1 + a_2X_2 + \cdots + a_nX_n) = a_1\,\mathrm{E}(X_1) + a_2\,\mathrm{E}(X_2) + \cdots + a_n\,\mathrm{E}(X_n)$	
	$\mathrm{Var}(a_1X_1 + a_2X_2 + \cdots + a_nX_n) = a_1^2\,\mathrm{Var}(X_1) + a_2^2\,\mathrm{Var}(X_2) + \cdots + a_n^2\,\mathrm{Var}(X_n)$	
for independent identically distributed variables $X_1, X_2, \ldots, X_n$	$\mathrm{E}(X_1 + X_2 + \cdots + X_n) = n\mu$	$\mathrm{Var}(X_1 + X_2 + \cdots + X_n) = n\sigma^2$
approximate confidence interval for μ	$\left(\bar{x} - z\frac{s}{\sqrt{n}},\ \bar{x} + z\frac{s}{\sqrt{n}}\right)$	
distribution of sample mean $\bar{X}$	mean $\mathrm{E}(\bar{X}) = \mu$	variance $\mathrm{Var}(\bar{X}) = \frac{\sigma^2}{n}$

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Circular (trigonometric) functions	
$\cos^2(x) + \sin^2(x) = 1$	
$1 + \tan^2(x) = \sec^2(x)$	$\cot^2(x) + 1 = \operatorname{cosec}^2(x)$
$\sin(x+y) = \sin(x)\cos(y) + \cos(x)\sin(y)$	$\sin(x-y) = \sin(x)\cos(y) - \cos(x)\sin(y)$
$\cos(x+y) = \cos(x)\cos(y) - \sin(x)\sin(y)$	$\cos(x-y) = \cos(x)\cos(y) + \sin(x)\sin(y)$
$\tan(x+y) = \dfrac{\tan(x)+\tan(y)}{1-\tan(x)\tan(y)}$	$\tan(x-y) = \dfrac{\tan(x)-\tan(y)}{1+\tan(x)\tan(y)}$
$\cos(2x) = \cos^2(x) - \sin^2(x) = 2\cos^2(x) - 1 = 1 - 2\sin^2(x)$	
$\sin(2x) = 2\sin(x)\cos(x)$	$\tan(2x) = \dfrac{2\tan(x)}{1-\tan^2(x)}$
$\sin^2(ax) = \dfrac{1}{2}(1-\cos(2ax))$	$\cos^2(ax) = \dfrac{1}{2}(1+\cos(2ax))$

Calculus	
$\dfrac{d}{dx}(x^n) = nx^{n-1}$	$\displaystyle\int x^n\,dx = \frac{1}{n+1}x^{n+1} + c,\ n \neq -1$
$\dfrac{d}{dx}(e^{ax}) = ae^{ax}$	$\displaystyle\int e^{ax}\,dx = \frac{1}{a}e^{ax} + c$
$\dfrac{d}{dx}(\log_e(x)) = \dfrac{1}{x}$	$\displaystyle\int \frac{1}{x}\,dx = \log_e\lvert x\rvert + c$
$\dfrac{d}{dx}(\sin(ax)) = a\cos(ax)$	$\displaystyle\int \sin(ax)\,dx = -\frac{1}{a}\cos(ax) + c$
$\dfrac{d}{dx}(\cos(ax)) = -a\sin(ax)$	$\displaystyle\int \cos(ax)\,dx = \frac{1}{a}\sin(ax) + c$
$\dfrac{d}{dx}(\tan(ax)) = a\sec^2(ax)$	$\displaystyle\int \sec^2(ax)\,dx = \frac{1}{a}\tan(ax) + c$
$\dfrac{d}{dx}(\cot(ax)) = -a\operatorname{cosec}^2(ax)$	$\displaystyle\int \operatorname{cosec}^2(ax)\,dx = -\frac{1}{a}\cot(ax) + c$
$\dfrac{d}{dx}(\sec(ax)) = a\sec(ax)\tan(ax)$	$\displaystyle\int \sec(ax)\tan(ax)\,dx = \frac{1}{a}\sec(ax) + c$
$\dfrac{d}{dx}(\operatorname{cosec}(ax)) = -a\operatorname{cosec}(ax)\cot(ax)$	$\displaystyle\int \operatorname{cosec}(ax)\cot(ax)\,dx = -\frac{1}{a}\operatorname{cosec}(ax) + c$
$\dfrac{d}{dx}\left(\sin^{-1}(ax)\right) = \dfrac{a}{\sqrt{1-(ax)^2}}$	$\displaystyle\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\left(\frac{x}{a}\right) + c,\ a > 0$
$\dfrac{d}{dx}\left(\cos^{-1}(ax)\right) = \dfrac{-a}{\sqrt{1-(ax)^2}}$	$\displaystyle\int \frac{-1}{\sqrt{a^2-x^2}}\,dx = \cos^{-1}\left(\frac{x}{a}\right) + c,\ a > 0$
$\dfrac{d}{dx}\left(\tan^{-1}(ax)\right) = \dfrac{a}{1+(ax)^2}$	$\displaystyle\int \frac{a}{a^2+x^2}\,dx = \tan^{-1}\left(\frac{x}{a}\right) + c$
$\displaystyle\int (ax+b)^{-1}\,dx = \frac{1}{a}\log_e\lvert ax+b\rvert + c$	$\displaystyle\int (ax+b)^n\,dx = \frac{1}{a(n+1)}(ax+b)^{n+1} + c,\ n \neq -1$

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Calculus — continued	
product rule	$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$
quotient rule	$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$
chain rule	$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$
integration by parts	$\int u\frac{dv}{dx}\,dx = uv - \int v\frac{du}{dx}\,dx$
Euler's method	If $\frac{dy}{dx} = f(x, y)$, $x_0 = a$ and $y_0 = b$, then $x_{n+1} = x_n + h$ and $y_{n+1} = y_n + hf(x_n, y_n)$.
arc length parametric	$\int_{t_1}^{t_2} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt$
surface area Cartesian about the x-axis	$\int_{x_1}^{x_2} 2\pi y\sqrt{1 + \left(\frac{dy}{dx}\right)^2}\,dx$
surface area Cartesian about the y-axis	$\int_{y_1}^{y_2} 2\pi x\sqrt{1 + \left(\frac{dx}{dy}\right)^2}\,dy$
surface area parametric about the x-axis	$\int_{t_1}^{t_2} 2\pi y\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt$
surface area parametric about the y-axis	$\int_{t_1}^{t_2} 2\pi x\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt$

Kinematics		
acceleration	$a = \frac{d^2x}{dt^2} = \frac{dv}{dt} = v\frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$	
constant acceleration formulas	$v = u + at$	$s = ut + \frac{1}{2}at^2$
	$v^2 = u^2 + 2as$	$s = \frac{1}{2}(u + v)t$

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Vectors in two and three dimensions		
	$\underset{\sim}{r}(t) = x(t)\underset{\sim}{i} + y(t)\underset{\sim}{j} + z(t)\underset{\sim}{k}$	$\lvert\underset{\sim}{r}(t)\rvert = \sqrt{x(t)^2 + y(t)^2 + z(t)^2}$
	$\dot{\underset{\sim}{r}}(t) = \frac{d\underset{\sim}{r}}{dt} = \frac{dx}{dt}\underset{\sim}{i} + \frac{dy}{dt}\underset{\sim}{j} + \frac{dz}{dt}\underset{\sim}{k}$	
for $\underset{\sim}{r}_1 = x_1\underset{\sim}{i} + y_1\underset{\sim}{j} + z_1\underset{\sim}{k}$ and $\underset{\sim}{r}_2 = x_2\underset{\sim}{i} + y_2\underset{\sim}{j} + z_2\underset{\sim}{k}$	vector scalar product $\underset{\sim}{r}_1 \cdot \underset{\sim}{r}_2 = \lvert\underset{\sim}{r}_1\rvert\lvert\underset{\sim}{r}_2\rvert\cos(\theta) = x_1x_2 + y_1y_2 + z_1z_2$	
	vector cross product $\underset{\sim}{r}_1 \times \underset{\sim}{r}_2 = \begin{vmatrix} \underset{\sim}{i} & \underset{\sim}{j} & \underset{\sim}{k} \\ x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \end{vmatrix} = (y_1z_2 - y_2z_1)\underset{\sim}{i} + (x_2z_1 - x_1z_2)\underset{\sim}{j} + (x_1y_2 - x_2y_1)\underset{\sim}{k}$	
vector equation of a line	$\underset{\sim}{r}(t) = \underset{\sim}{r}_1 + t\underset{\sim}{r}_2 = (x_1 + x_2t)\underset{\sim}{i} + (y_1 + y_2t)\underset{\sim}{j} + (z_1 + z_2t)\underset{\sim}{k}$	
parametric equation of a line	$x(t) = x_1 + x_2t \quad y(t) = y_1 + y_2t \quad z(t) = z_1 + z_2t$	
vector equation of a plane	$\underset{\sim}{r}(s,t) = \underset{\sim}{r}_0 + s\underset{\sim}{r}_1 + t\underset{\sim}{r}_2 = (x_0 + x_1s + x_2t)\underset{\sim}{i} + (y_0 + y_1s + y_2t)\underset{\sim}{j} + (z_0 + z_1s + z_2t)\underset{\sim}{k}$	
parametric equation of a plane	$x(s,t) = x_0 + x_1s + x_2t \quad y(s,t) = y_0 + y_1s + y_2t \quad z(s,t) = z_0 + z_1s + z_2t$	
Cartesian equation of a plane	$ax + by + cz = d$	

END OF FORMULA SHEET

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6.

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9.

10.

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5 questions locked, up to question 10.

PREVIEW

That’s the end of the preview

You’ve seen the first 5 of the 10 questions in Specialist Mathematics Unit 3 & 4 — Examination 1 (Practice 3). The full paper is worth 40 marks.

STILL TO COME	QUESTIONS	MARKS
Examination 1 — technology-free	5	21

The full paper comes with

- Every question, printed exactly like the ones in this preview
- The complete answer key, with the working behind every mark
- A topic report after marking, showing what to practice next

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