Specialist Mathematics Unit 3 & 4 — Examination 2 (Practice 3) — Answers & Explanations

Answers to the 13 questions in the free preview.

Section A — multiple choice

1. B. For all $x \in \mathbb{Z}$, $x^2 \geq x$.
For an integer, $x^2 - x = x(x-1)$ is a product of consecutive integers, never negative, so B is true and C is false. A fails at $x = \frac{1}{2}$; D fails at $x = -2$.

2. D. if n is even, then n^2 is even
The contrapositive of "if P then Q" is "if not Q then not P": if n is even (not odd), then n^2 is even (not odd). A is the converse; C describes a proof by contradiction.

3. A. $\dfrac{3}{x+1} + \dfrac{2}{(x+1)^2}$
$3x + 5 = 3(x+1) + 2$, so dividing by $(x+1)^2$ gives $\frac{3}{x+1} + \frac{2}{(x+1)^2}$. A repeated factor needs both $\frac{A}{x+1}$ and $\frac{B}{(x+1)^2}$.

4. C. $y = 2x$
$2x^3 - x + 1 = 2x(x^2+1) - 3x + 1$, so $y = 2x + \dfrac{1-3x}{x^2+1}$, and the fraction tends to 0 as $x \to \pm\infty$. The graph has no vertical asymptote since $x^2 + 1 > 0$.

5. A. -4
$(1+i)^2 = 2i$, so $(1+i)^8 = (2i)^4 = 16$. $(1-i)^2 = -2i$, so $(1-i)^4 = (-2i)^2 = -4$. The quotient is -4. (In polar form: $\frac{16\,\mathrm{cis}(2\pi)}{4\,\mathrm{cis}(-\pi)} = 4\,\mathrm{cis}(3\pi) = -4$.)

6. D. the circle $|z| = 1$
The quadratic formula gives $z = \cos(\theta) \pm \sqrt{\cos^2(\theta) - 1} = \cos(\theta) \pm i\sin(\theta) = \mathrm{cis}(\pm\theta)$, each of modulus 1. (Also, the product of the roots is 1 and they are conjugates.)

7. B. the ray starting at 1, excluding 1, at an angle of $\frac{\pi}{4}$ to the positive real axis
$z - 1$ is the vector from 1 to z; its argument is $\frac{\pi}{4}$ for points on the ray from 1 in that direction. $\mathrm{Arg}(0)$ is undefined, so $z = 1$ is excluded. C would also contain the points with $\mathrm{Arg}(z-1) = -\frac{3\pi}{4}$.

8. C. 4.00
Each pass applies Euler's method to $\frac{dy}{dx} = \frac{y}{x}$ with $h = 0.25$: $y = 2.5, 3, 3.5, 4$ at $x = 1.25, 1.5, 1.75, 2$, when the loop stops. The exact solution through $(1, 2)$ is $y = 2x$, a straight line, so every Euler step is exact. 3.50 stops one pass early.

9. D. 3
This is a logistic equation with carrying capacity 3: for $0 < y < 3$, $\frac{dy}{dx} > 0$ and y increases towards the equilibrium $y = 3$. 1.5 is where y grows fastest.

10. D. 13 m
The particle stops and turns at $t = 2$. Distance $= \int_0^2 (4 - 2t)\,dt + \left|\int_2^5 (4-2t)\,dt\right| = 4 + 9 = 13$. -5 m is the displacement $\int_0^5 (4-2t)\,dt$.

11. B. 3
$\frac{d}{dx}\left(\frac{1}{2}v^2\right) = -4x$ gives $v^2 = 36 - 4x^2$, which is zero when $x = \pm 3$.

12. B. $\log_e\left(\frac{4}{3}\right)$
With $u = 1 + \cos(x)$, $du = -\sin(x)\,dx$: the antiderivative is $-\log_e(1+\cos(x))$, giving $-\log_e\left(\frac{3}{2}\right) + \log_e(2) = \log_e\left(\frac{4}{3}\right)$. A has the sign reversed.

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13. B. 1

Integrate with respect to y: $x = \sin(y)$, so the area is $\int_0^{\frac{\pi}{2}} \sin(y)\, dy = 1$. $\frac{\pi}{2} - 1$ is the area between the curve and the x-axis for $0 \leq x \leq 1$; the two add to the rectangle $1 \times \frac{\pi}{2}$.

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PREVIEW

That’s the end of the preview answers

These are the answers to the 13 preview questions. The full answer key covers all 26 questions in Specialist Mathematics Unit 3 & 4 — Examination 2 (Practice 3), with the working behind every mark.

STILL TO COME	QUESTIONS	MARKS
Section A — multiple choice	7	7
Section B — extended response	6	60

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