Specialist Mathematics Unit 3 & 4 — Examination 1 (Practice 4) — Answers & Explanations

Answers to the 5 questions in the free preview.

Examination 1 — technology-free

1. (3 marks in total)

True for $n = 1$: $f'(x) = (1 - x)e^{-x} = (-1)(x - 1)e^{-x}$. If $f^{(k)}(x) = (-1)^k(x - k)e^{-x}$, differentiating gives $(-1)^{k+1}(x - (k + 1))e^{-x}$.

[3 marks]

Base case: $f'(x) = e^{-x} - xe^{-x} = -(x - 1)e^{-x}$, which is the formula with $n = 1$. Inductive step: assume $f^{(k)}(x) = (-1)^k(x - k)e^{-x}$. Then by the product rule

$$f^{(k+1)}(x) = (-1)^k\left[e^{-x} - (x - k)e^{-x}\right] = (-1)^k(k + 1 - x)e^{-x} = (-1)^{k+1}\big(x - (k + 1)\big)e^{-x},$$

the formula with $n = k + 1$. By induction it holds for all $n \in \mathbb{N}$. Marks: 1 base case; 1 differentiating the assumed form; 1 rearranging to the $n = k + 1$ form and concluding.

2. (4 marks in total)

a. $z^n + z^{-n} = \text{cis}(n\theta) + \text{cis}(-n\theta) = 2\cos(n\theta)$

[1 mark]

By de Moivre's theorem; the imaginary parts $\pm\sin(n\theta)$ cancel.

b. $\left(z + z^{-1}\right)^3 = \left(z^3 + z^{-3}\right) + 3\left(z + z^{-1}\right)$, so $8\cos^3(\theta) = 2\cos(3\theta) + 6\cos(\theta)$.

[3 marks]

Binomial expansion: $(z + z^{-1})^3 = z^3 + 3z + 3z^{-1} + z^{-3}$. Grouping and using part a: $(2\cos(\theta))^3 = 2\cos(3\theta) + 3 \times 2\cos(\theta)$. Divide by 8. Marks: 1 expansion; 1 grouping into $z^n + z^{-n}$ pairs; 1 result.

3. (4 marks in total)

a. $x = 1$ and $y = 1$

[1 mark]

The denominator is zero at $x = 1$ (the numerator is not), and $f(x) = \left(\frac{x}{x-1}\right)^2 \to 1$ as $x \to \pm\infty$.

b. $f'(x) = \dfrac{2x(x-1)^2 - 2x^2(x-1)}{(x-1)^4} = -\dfrac{2x}{(x-1)^3}$; stationary point $(0, 0)$.

[2 marks]

Cancel a factor of $x - 1$: the numerator becomes $2x(x - 1) - 2x^2 = -2x$. $f'(x) = 0$ only at $x = 0$. 1 mark for the derivative, 1 mark for the point.

c. $\left(-\dfrac{1}{2}, \dfrac{1}{9}\right)$

[1 mark]

$f''(x) = 0$ at $x = -\frac{1}{2}$, and f'' changes sign there (the denominator is positive). $f\left(-\frac{1}{2}\right) = \frac{1/4}{9/4} = \frac{1}{9}$.

4. (3 marks in total)

$\dfrac{52}{3}$

[3 marks]

$\dot{x} = 2t$, $\dot{y} = 2t^2$, so $\sqrt{\dot{x}^2 + \dot{y}^2} = \sqrt{4t^2(1 + t^2)} = 2t\sqrt{1 + t^2}$ for $t \geq 0$.

$$L = \int_0^{2\sqrt{2}} 2t\sqrt{1 + t^2}\, dt = \left[\frac{2}{3}(1 + t^2)^{\frac{3}{2}}\right]_0^{2\sqrt{2}} = \frac{2}{3}(27 - 1) = \frac{52}{3}.$$

Marks: 1 for the integrand; 1 for the antiderivative; 1 for the value.

PrepNest

5. (4 marks in total)

a. $-\frac{1}{8}$ m s⁻²

[2 marks]

$a = v\frac{dv}{dx} = \frac{1}{x+1} \times \frac{-1}{(x+1)^2} = -\frac{1}{(x+1)^3}$, which is $-\frac{1}{8}$ at $x = 1$. 1 mark for $a = v\frac{dv}{dx}$, 1 mark for the value.

b. $\frac{15}{2}$ s

[2 marks]

$\frac{dt}{dx} = \frac{1}{v} = x + 1$, so $t = \int_0^3 (x+1)\,dx = \frac{9}{2} + 3 = \frac{15}{2}$. 1 mark for $\frac{dt}{dx} = \frac{1}{v}$, 1 mark for the time.

PrepNest

PREVIEW

That’s the end of the preview answers

These are the answers to the 5 preview questions. The full answer key covers all 10 questions in Specialist Mathematics Unit 3 & 4 — Examination 1 (Practice 4), with the working behind every mark.

STILL TO COME	QUESTIONS	MARKS
Examination 1 — technology-free	5	22

The full paper comes with

- Every question, printed exactly like the ones in this preview
- The complete answer key, with the working behind every mark
- A topic report after marking, showing what to practice next

Get the full paper at
prepnest.com.au/practice/exams/specialist_maths-year_12-4-exam1

PrepNest