Specialist Mathematics Unit 3 & 4 — Examination 2 (Practice 4) — Answers & Explanations

Answers to the 13 questions in the free preview.

Section A — multiple choice

1. D. If $x = 2$, then $x^2 = 4$.
The contrapositive of "if P then Q" is "if not Q then not P": if $x = 2$, then $x^2 = 4$. B is the inverse (and is false, since x could be -2); A is the converse.

2. A. $6^{k+1} - 1 = 6(6^k - 1) + 5$
$6(6^k - 1) + 5 = 6^{k+1} - 6 + 5 = 6^{k+1} - 1$, and the right side is $6 \times 5m + 5 = 5(6m + 1)$. B and C are not identities; D would need $6^{k+1} - 1 = 5 \times 6^k$, which is false.

3. B. $(-\infty, 1]$
On $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, $\cos(x) \in (0, 1]$, so $\sec(x) \geq 1$ and $3 - 2\sec(x) \leq 1$, with no lower bound. C is the range over the whole maximal domain.

4. C. $[-1, 1)$
$y = 1 - \dfrac{2}{x^2 + 1}$. The fraction $\frac{2}{x^2+1}$ takes every value in $(0, 2]$, so $y \in [-1, 1)$: $y = -1$ at $x = 0$, and $y = 1$ is an asymptote that is never reached.

5. D. -64
$\sqrt{3} + i = 2\,\mathrm{cis}\left(\frac{\pi}{6}\right)$, so the sixth power is $64\,\mathrm{cis}(\pi) = -64$.

6. B. 9
The other solution is the conjugate $2 + 3i$. Sum of roots $= 4 = -a$, so $a = -4$; product $= 4 + 9 = 13 = b$. So $a + b = 9$. 17 takes $a = 4$.

7. C. $\{z : |z| \leq 2\} \cap \{z : \mathrm{Im}(z) \geq 1\}$
The region is the part of the disc of radius 2 centred at the origin that lies on or above the horizontal line $\mathrm{Im}(z) = 1$. A uses a vertical line; D is a disc centred at i.

8. A. $\dfrac{dy}{dx} = y(2 - y)$
The gradients depend only on y (each row of segments is parallel), which rules out B. They are zero along $y = 0$ and $y = 2$, positive for $0 < y < 2$ and negative outside that interval: $y(2 - y)$. C has the opposite signs; D is zero only on $y = 2$.

9. C. 25.85
$50 = 20 + 60e^{-10k}$ gives $e^{-10k} = \frac{1}{2}$, so $k = \frac{\log_e(2)}{10}$. Then $60e^{-kt} = 10$ gives $t = \frac{\log_e(6)}{k} = \frac{10\log_e(6)}{\log_e(2)} \approx 25.85$. The excess temperature halves every 10 minutes, so after 20 minutes it is 35 °C and after 30 minutes 27.5 °C — the answer lies between. 17.92 is $10\log_e(6)$.

10. A. 6
$v = 3t^2 - 4t + 3$ and $x = t^3 - 2t^2 + 3t$, so $x(2) = 8 - 8 + 6 = 6$. 7 is the velocity at $t = 2$; 14 omits the -4 in the acceleration.

11. A. $\dfrac{3}{4}$
$y = x^{\frac{3}{2}}$, so $\frac{dy}{dx} = \frac{3}{2}x^{\frac{1}{2}}$ and $\frac{d^2y}{dx^2} = \frac{3}{4}x^{-\frac{1}{2}} = \frac{3}{4}$ at $x = 1$. $\frac{3}{2}$ is the first derivative.

12. C. 0.80
$\displaystyle\int_0^3 xe^{-x}\,dx = \Big[-(x+1)e^{-x}\Big]_0^3 = 1 - 4e^{-3} \approx 0.80$. 1.00 is the area for $x \geq 0$ without the line $x = 3$.

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13. D. 13.18

For $0 \le y \le 1$ the region runs from $x = e^y$ to $x = e$, so $V = \pi \displaystyle\int_0^1 \left(e^2 - e^{2y}\right) dy = \frac{\pi(e^2+1)}{2} \approx 13.18$. 10.04 uses only the inner radius; 2.26 rotates about the x-axis; 7.39 is e^2.

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PREVIEW

That’s the end of the preview answers

These are the answers to the 13 preview questions. The full answer key covers all 26 questions in Specialist Mathematics Unit 3 & 4 — Examination 2 (Practice 4), with the working behind every mark.

STILL TO COME	QUESTIONS	MARKS
Section A — multiple choice	7	7
Section B — extended response	6	60

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- Every question, printed exactly like the ones in this preview
- The complete answer key, with the working behind every mark
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