EXAM PAPER

Specialist Mathematics Unit 3 & 4 — Examination 2 (Practice 4)

PREVIEW
This file has the first 13 of the 26 questions in this paper. The rest is locked: it comes with a PrepNest plan, or with the paper itself for VCE.

Reading time: 15 minutes (no writing)
Writing time: 120 minutes
Total time: 135 minutes

SECTIONS IN THIS PAPER

- Section A — multiple choice — 45 min (calculator allowed) — 13 questions, 13 marks
- Section B — extended response — 75 min (calculator allowed) — 0 questions, 0 marks

A formula sheet is printed at the end of this paper.

INSTRUCTIONS

You are not permitted to write during reading time — you may only read the paper and plan your approach. Answer every question you can. Write your working in the space provided for long-answer questions. Marking guidance and full explanations are provided in the separate answer key.

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Section A — multiple choice

45 minutes • Calculator allowed

INSTRUCTIONS

- Answer all questions. Choose the response that is correct for the question.
- A correct answer scores 1; an incorrect answer scores 0. Marks will not be deducted for incorrect answers.
- Unless otherwise indicated, the diagrams in this book are not drawn to scale.
- Take the acceleration due to gravity to have magnitude g m s^{-2}, where g = 9.8.

Question 1

The contrapositive of the statement 'If $x^2 \neq 4$, then $x \neq 2$' is

A. If $x \neq 2$, then $x^2 \neq 4$.

B. If $x^2 = 4$, then $x = 2$.

C. If $x^2 \neq 4$, then $x = 2$.

D. If $x = 2$, then $x^2 = 4$.

Question 2

A student proves by induction that $6^n - 1$ is divisible by 5 for all $n \in \mathbb{N}$. Assuming that $6^k - 1 = 5m$ for some integer m, a correct inductive step uses

A. $6^{k+1} - 1 = 6(6^k - 1) + 5$

B. $6^{k+1} - 1 = (6^k - 1) + 5$

C. $6^{k+1} - 1 = 6(6^k - 1) - 5$

D. $6^{k+1} - 1 = 6^k(6 - 1)$

Question 3

The range of the function $f : \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \to \mathbb{R}$, $f(x) = 3 - 2\sec(x)$, is

A. $[1, \infty)$

B. $(-\infty, 1]$

C. $(-\infty, 1] \cup [5, \infty)$

D. $[-1, 1]$

Question 4

The range of the function with rule $y = \dfrac{x^2 - 1}{x^2 + 1}$ is

A. $(-1, 1)$

B. $[-1, 1]$

C. $[-1, 1)$

D. $(-\infty, 1)$

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Question 5

$\left(\sqrt{3}+i\right)^6$ is equal to

A. 64

B. $-64i$

C. $64i$

D. -64

Question 6

The quadratic equation $z^2+az+b=0$, where $a,b\in\mathbb{R}$, has $z=2-3i$ as a solution. The value of $a+b$ is

A. 5

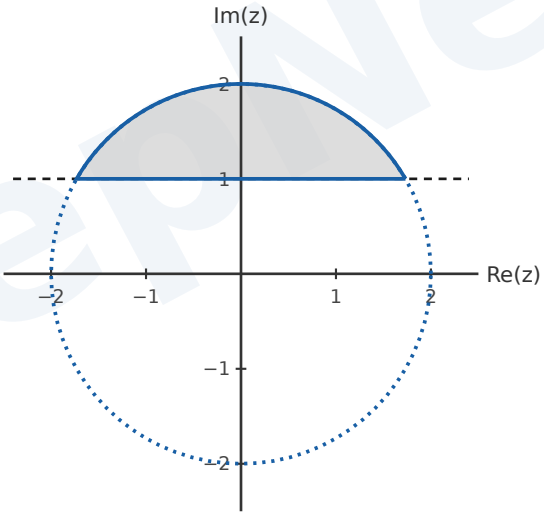
B. 9

C. 13

D. 17

Question 7

Which one of the following sets is represented by the shaded region of the complex plane shown below? The boundary is included.



A. $\{z:|z|\leq 2\}\cap\{z:\mathrm{Re}(z)\geq 1\}$

B. $\{z:|z|\geq 2\}\cap\{z:\mathrm{Im}(z)\geq 1\}$

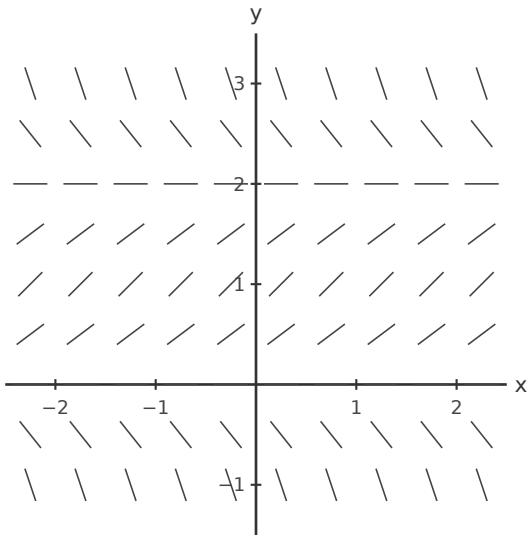
C. $\{z:|z|\leq 2\}\cap\{z:\mathrm{Im}(z)\geq 1\}$

D. $\{z:|z-i|\leq 2\}\cap\{z:\mathrm{Im}(z)\geq 1\}$

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Question 8

Which one of the following differential equations has the slope field shown below?



A. $\dfrac{dy}{dx} = y(2 - y)$

B. $\dfrac{dy}{dx} = x(2 - x)$

C. $\dfrac{dy}{dx} = y(y - 2)$

D. $\dfrac{dy}{dx} = 2 - y$

Question 9

A metal rod cools according to Newton's law of cooling. Its temperature T °C after t minutes is $T = 20 + 60e^{-kt}$, and $T = 50$ when $t = 10$. The time, in minutes correct to two decimal places, taken for the rod to cool to 30 °C is

A. 17.92

B. 20.00

C. 25.85

D. 30.00

Question 10

A particle moves in a straight line with acceleration $a = 6t - 4$ m s^{-2} at time t seconds. Initially it is at the origin with velocity 3 m s^{-1}. Its displacement from the origin, in metres, when $t = 2$ is

A. 6

B. 7

C. 8

D. 14

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Question 11

If $y^2 = x^3$ and $y > 0$, then $\dfrac{d^2y}{dx^2}$ at the point $(1, 1)$ is

A. $\dfrac{3}{4}$

B. $\dfrac{3}{2}$

C. $\dfrac{1}{4}$

D. $-\dfrac{3}{4}$

Question 12

The area of the region bounded by the graph of $y = xe^{-x}$, the x-axis and the line $x = 3$, correct to two decimal places, is

A. 0.05

B. 0.20

C. 0.80

D. 1.00

Question 13

The region bounded by the graph of $y = \log_e(x)$, the x-axis and the line $x = e$ is rotated about the y-axis. The volume of the solid formed, correct to two decimal places, is

A. 2.26

B. 7.39

C. 10.04

D. 13.18

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FORMULA SHEET

Specialist Mathematics formulas

Mensuration	
area of a circle segment	$\frac{r^2}{2}(\theta - \sin(\theta))$
volume of a cylinder	$\pi r^2 h$
volume of a cone	$\frac{1}{3}\pi r^2 h$
volume of a pyramid	$\frac{1}{3}Ah$
volume of a sphere	$\frac{4}{3}\pi r^3$
area of a triangle	$\frac{1}{2}bc\sin(A)$
sine rule	$\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}$
cosine rule	$c^2 = a^2 + b^2 - 2ab\cos(C)$

Algebra, number and structure (complex numbers)	
$z = x + iy = r(\cos(\theta) + i\sin(\theta)) = r\,\mathrm{cis}(\theta)$	$\lvert z\rvert = \sqrt{x^2 + y^2} = r$
$-\pi < \mathrm{Arg}(z) \leq \pi$	$z_1 z_2 = r_1 r_2\,\mathrm{cis}(\theta_1 + \theta_2)$
$\frac{z_1}{z_2} = \frac{r_1}{r_2}\mathrm{cis}(\theta_1 - \theta_2)$	de Moivre's theorem $z^n = r^n\,\mathrm{cis}(n\theta)$

Data analysis, probability and statistics		
for independent random variables $X_1, X_2, \ldots, X_n$	$\mathrm{E}(aX_1 + b) = a\,\mathrm{E}(X_1) + b$	$\mathrm{Var}(aX_1 + b) = a^2\,\mathrm{Var}(X_1)$
	$\mathrm{E}(a_1X_1 + a_2X_2 + \cdots + a_nX_n) = a_1\,\mathrm{E}(X_1) + a_2\,\mathrm{E}(X_2) + \cdots + a_n\,\mathrm{E}(X_n)$	
	$\mathrm{Var}(a_1X_1 + a_2X_2 + \cdots + a_nX_n) = a_1^2\,\mathrm{Var}(X_1) + a_2^2\,\mathrm{Var}(X_2) + \cdots + a_n^2\,\mathrm{Var}(X_n)$	
for independent identically distributed variables $X_1, X_2, \ldots, X_n$	$\mathrm{E}(X_1 + X_2 + \cdots + X_n) = n\mu$	$\mathrm{Var}(X_1 + X_2 + \cdots + X_n) = n\sigma^2$
approximate confidence interval for μ	$\left(\bar{x} - z\frac{s}{\sqrt{n}},\ \bar{x} + z\frac{s}{\sqrt{n}}\right)$	
distribution of sample mean $\bar{X}$	mean $\mathrm{E}(\bar{X}) = \mu$	variance $\mathrm{Var}(\bar{X}) = \frac{\sigma^2}{n}$

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Circular (trigonometric) functions	
$\cos^2(x) + \sin^2(x) = 1$	
$1 + \tan^2(x) = \sec^2(x)$	$\cot^2(x) + 1 = \operatorname{cosec}^2(x)$
$\sin(x+y) = \sin(x)\cos(y) + \cos(x)\sin(y)$	$\sin(x-y) = \sin(x)\cos(y) - \cos(x)\sin(y)$
$\cos(x+y) = \cos(x)\cos(y) - \sin(x)\sin(y)$	$\cos(x-y) = \cos(x)\cos(y) + \sin(x)\sin(y)$
$\tan(x+y) = \dfrac{\tan(x)+\tan(y)}{1-\tan(x)\tan(y)}$	$\tan(x-y) = \dfrac{\tan(x)-\tan(y)}{1+\tan(x)\tan(y)}$
$\cos(2x) = \cos^2(x) - \sin^2(x) = 2\cos^2(x) - 1 = 1 - 2\sin^2(x)$	
$\sin(2x) = 2\sin(x)\cos(x)$	$\tan(2x) = \dfrac{2\tan(x)}{1-\tan^2(x)}$
$\sin^2(ax) = \dfrac{1}{2}(1-\cos(2ax))$	$\cos^2(ax) = \dfrac{1}{2}(1+\cos(2ax))$

Calculus	
$\dfrac{d}{dx}(x^n) = nx^{n-1}$	$\displaystyle\int x^n\,dx = \frac{1}{n+1}x^{n+1} + c,\ n \neq -1$
$\dfrac{d}{dx}(e^{ax}) = ae^{ax}$	$\displaystyle\int e^{ax}\,dx = \frac{1}{a}e^{ax} + c$
$\dfrac{d}{dx}(\log_e(x)) = \dfrac{1}{x}$	$\displaystyle\int \frac{1}{x}\,dx = \log_e\lvert x\rvert + c$
$\dfrac{d}{dx}(\sin(ax)) = a\cos(ax)$	$\displaystyle\int \sin(ax)\,dx = -\frac{1}{a}\cos(ax) + c$
$\dfrac{d}{dx}(\cos(ax)) = -a\sin(ax)$	$\displaystyle\int \cos(ax)\,dx = \frac{1}{a}\sin(ax) + c$
$\dfrac{d}{dx}(\tan(ax)) = a\sec^2(ax)$	$\displaystyle\int \sec^2(ax)\,dx = \frac{1}{a}\tan(ax) + c$
$\dfrac{d}{dx}(\cot(ax)) = -a\operatorname{cosec}^2(ax)$	$\displaystyle\int \operatorname{cosec}^2(ax)\,dx = -\frac{1}{a}\cot(ax) + c$
$\dfrac{d}{dx}(\sec(ax)) = a\sec(ax)\tan(ax)$	$\displaystyle\int \sec(ax)\tan(ax)\,dx = \frac{1}{a}\sec(ax) + c$
$\dfrac{d}{dx}(\operatorname{cosec}(ax)) = -a\operatorname{cosec}(ax)\cot(ax)$	$\displaystyle\int \operatorname{cosec}(ax)\cot(ax)\,dx = -\frac{1}{a}\operatorname{cosec}(ax) + c$
$\dfrac{d}{dx}\left(\sin^{-1}(ax)\right) = \dfrac{a}{\sqrt{1-(ax)^2}}$	$\displaystyle\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\left(\frac{x}{a}\right) + c,\ a > 0$
$\dfrac{d}{dx}\left(\cos^{-1}(ax)\right) = \dfrac{-a}{\sqrt{1-(ax)^2}}$	$\displaystyle\int \frac{-1}{\sqrt{a^2-x^2}}\,dx = \cos^{-1}\left(\frac{x}{a}\right) + c,\ a > 0$
$\dfrac{d}{dx}\left(\tan^{-1}(ax)\right) = \dfrac{a}{1+(ax)^2}$	$\displaystyle\int \frac{a}{a^2+x^2}\,dx = \tan^{-1}\left(\frac{x}{a}\right) + c$
$\displaystyle\int (ax+b)^{-1}\,dx = \frac{1}{a}\log_e\lvert ax+b\rvert + c$	$\displaystyle\int (ax+b)^n\,dx = \frac{1}{a(n+1)}(ax+b)^{n+1} + c,\ n \neq -1$

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Calculus — continued	
product rule	$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$
quotient rule	$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$
chain rule	$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$
integration by parts	$\int u\frac{dv}{dx}\,dx = uv - \int v\frac{du}{dx}\,dx$
Euler's method	If $\frac{dy}{dx} = f(x, y)$, $x_0 = a$ and $y_0 = b$, then $x_{n+1} = x_n + h$ and $y_{n+1} = y_n + hf(x_n, y_n)$.
arc length parametric	$\int_{t_1}^{t_2} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt$
surface area Cartesian about the x-axis	$\int_{x_1}^{x_2} 2\pi y\sqrt{1 + \left(\frac{dy}{dx}\right)^2}\,dx$
surface area Cartesian about the y-axis	$\int_{y_1}^{y_2} 2\pi x\sqrt{1 + \left(\frac{dx}{dy}\right)^2}\,dy$
surface area parametric about the x-axis	$\int_{t_1}^{t_2} 2\pi y\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt$
surface area parametric about the y-axis	$\int_{t_1}^{t_2} 2\pi x\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt$

Kinematics		
acceleration	$a = \frac{d^2x}{dt^2} = \frac{dv}{dt} = v\frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$	
constant acceleration formulas	$v = u + at$	$s = ut + \frac{1}{2}at^2$
	$v^2 = u^2 + 2as$	$s = \frac{1}{2}(u + v)t$

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Vectors in two and three dimensions		
	$\underset{\sim}{r}(t) = x(t)\underset{\sim}{i} + y(t)\underset{\sim}{j} + z(t)\underset{\sim}{k}$	$\lvert\underset{\sim}{r}(t)\rvert = \sqrt{x(t)^2 + y(t)^2 + z(t)^2}$
	$\dot{\underset{\sim}{r}}(t) = \frac{d\underset{\sim}{r}}{dt} = \frac{dx}{dt}\underset{\sim}{i} + \frac{dy}{dt}\underset{\sim}{j} + \frac{dz}{dt}\underset{\sim}{k}$	
for $\underset{\sim}{r}_1 = x_1\underset{\sim}{i} + y_1\underset{\sim}{j} + z_1\underset{\sim}{k}$ and $\underset{\sim}{r}_2 = x_2\underset{\sim}{i} + y_2\underset{\sim}{j} + z_2\underset{\sim}{k}$	vector scalar product $\underset{\sim}{r}_1 \cdot \underset{\sim}{r}_2 = \lvert\underset{\sim}{r}_1\rvert\lvert\underset{\sim}{r}_2\rvert\cos(\theta) = x_1x_2 + y_1y_2 + z_1z_2$	
	vector cross product $\underset{\sim}{r}_1 \times \underset{\sim}{r}_2 = \begin{vmatrix} \underset{\sim}{i} & \underset{\sim}{j} & \underset{\sim}{k} \\ x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \end{vmatrix} = (y_1z_2 - y_2z_1)\underset{\sim}{i} + (x_2z_1 - x_1z_2)\underset{\sim}{j} + (x_1y_2 - x_2y_1)\underset{\sim}{k}$	
vector equation of a line	$\underset{\sim}{r}(t) = \underset{\sim}{r}_1 + t\underset{\sim}{r}_2 = (x_1 + x_2t)\underset{\sim}{i} + (y_1 + y_2t)\underset{\sim}{j} + (z_1 + z_2t)\underset{\sim}{k}$	
parametric equation of a line	$x(t) = x_1 + x_2t \quad y(t) = y_1 + y_2t \quad z(t) = z_1 + z_2t$	
vector equation of a plane	$\underset{\sim}{r}(s,t) = \underset{\sim}{r}_0 + s\underset{\sim}{r}_1 + t\underset{\sim}{r}_2 = (x_0 + x_1s + x_2t)\underset{\sim}{i} + (y_0 + y_1s + y_2t)\underset{\sim}{j} + (z_0 + z_1s + z_2t)\underset{\sim}{k}$	
parametric equation of a plane	$x(s,t) = x_0 + x_1s + x_2t \quad y(s,t) = y_0 + y_1s + y_2t \quad z(s,t) = z_0 + z_1s + z_2t$	
Cartesian equation of a plane	$ax + by + cz = d$	

END OF FORMULA SHEET

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6.

13 questions locked, up to question 6.

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PREVIEW

That’s the end of the preview

You’ve seen the first 13 of the 26 questions in Specialist Mathematics Unit 3 & 4 — Examination 2 (Practice 4). The full paper is worth 80 marks.

STILL TO COME	QUESTIONS	MARKS
Section A — multiple choice	7	7
Section B — extended response	6	60

The full paper comes with

- Every question, printed exactly like the ones in this preview
- The complete answer key, with the working behind every mark
- A topic report after marking, showing what to practice next

Get the full paper at
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