Specialist Mathematics Unit 3 & 4 — Examination 1 (Practice 5) — Answers & Explanations

Answers to the 5 questions in the free preview.

Examination 1 — technology-free

1. (3 marks in total)

Assume $m^2 - n^2 = (m-n)(m+n) = 10$. The factors $m-n$ and $m+n$ have the same parity, so their product is odd or a multiple of 4 — never 10.

[3 marks]

Suppose, for a contradiction, that positive integers m and n satisfy $m^2 - n^2 = 10$. Then $(m-n)(m+n) = 10$. Since $(m+n) - (m-n) = 2n$ is even, $m-n$ and $m+n$ are both even or both odd. If both are odd, the product is odd; if both are even, the product is a multiple of 4. But 10 is even and not a multiple of 4. This contradiction shows no such integers exist. Marks: 1 for the assumption and factorisation; 1 for the parity argument; 1 for the contradiction and conclusion.

2. (4 marks in total)

a. $(1+2i)^2 - (3+i)(1+2i) + 4 + 3i = (-3+4i) - (1+7i) + 4 + 3i = 0$

[1 mark]

Expand $(1+2i)^2 = -3+4i$ and $(3+i)(1+2i) = 3+6i+i-2 = 1+7i$.

b. $z = 2 - i$

[1 mark]

The sum of the solutions is $3+i$, so the other is $3+i-(1+2i) = 2-i$. (The coefficients are not all real, so the conjugate $1-2i$ is not a solution.)

c. $\overrightarrow{OA} \cdot \overrightarrow{OB} = (1)(2) + (2)(-1) = 0$; area $\dfrac{5}{2}$

[2 marks]

Treat $1+2i$ and $2-i$ as the vectors $(1,2)$ and $(2,-1)$. Their scalar product is zero, and each has length $\sqrt{5}$, so the area is $\frac{1}{2}\sqrt{5}\sqrt{5}$. (Equivalently, $2-i = -i(1+2i)$: a rotation by $-\frac{\pi}{2}$.) 1 mark each.

3. (4 marks in total)

a. $x^2 - y^2 = \sec^2(t) - \tan^2(t) = 1$

[1 mark]

Use the identity $1 + \tan^2(t) = \sec^2(t)$.

b. $[1, \infty)$

[1 mark]

For $-\frac{\pi}{2} < t < \frac{\pi}{2}$, $\cos(t) \in (0, 1]$, so $\sec(t) \geq 1$. The curve is only the right-hand branch of the hyperbola.

c. $\dfrac{dy}{dx} = \dfrac{1}{\sin(t)}$; gradient $\sqrt{2}$

[2 marks]

$\frac{dy}{dt} = \sec^2(t)$ and $\frac{dx}{dt} = \sec(t)\tan(t)$, so $\frac{dy}{dx} = \frac{\sec(t)}{\tan(t)} = \frac{1}{\sin(t)}$. At $t = \frac{\pi}{4}$ this is $\sqrt{2}$. 1 mark for the derivative, 1 mark for the gradient.

4. (4 marks in total)

a. $-\dfrac{1}{4}$

[2 marks]

$3x^2 + 3y^2\frac{dy}{dx} = 0$, so $\frac{dy}{dx} = -\frac{x^2}{y^2} = -\frac{1}{4}$. 1 mark for differentiating implicitly, 1 mark for the value.

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b. $-\dfrac{9}{16}$

[2 marks]

Differentiate $\frac{dy}{dx} = -\frac{x^2}{y^2}$ with the quotient rule: $\frac{d^2y}{dx^2} = -\frac{2xy^2 - 2x^2y\frac{dy}{dx}}{y^4}$. At $(1, 2)$ with $\frac{dy}{dx} = -\frac{1}{4}$: $-\frac{8+1}{16} = -\frac{9}{16}$. 1 mark for the derivative (including $\frac{dy}{dx}$ from the chain rule), 1 mark for the value.

5. (3 marks in total)

$\dfrac{8}{3}$

[3 marks]

With $u = x + 1$, $x = u - 1$, $du = dx$, and the terminals become 1 and 4:

$$\int_1^4 \frac{u-1}{\sqrt{u}}\,du = \int_1^4 \left(u^{\frac{1}{2}} - u^{-\frac{1}{2}}\right)du = \left[\frac{2}{3}u^{\frac{3}{2}} - 2u^{\frac{1}{2}}\right]_1^4 = \left(\frac{16}{3} - 4\right) - \left(\frac{2}{3} - 2\right) = \frac{8}{3}.$$

Marks: 1 for the integral in u with new terminals; 1 for the antiderivative; 1 for the value.

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PREVIEW

That’s the end of the preview answers

These are the answers to the 5 preview questions. The full answer key covers all 10 questions in Specialist Mathematics Unit 3 & 4 — Examination 1 (Practice 5), with the working behind every mark.

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