Specialist Mathematics Unit 3 & 4 — Examination 2 (Practice 5) — Answers & Explanations

Answers to the 13 questions in the free preview.

Section A — multiple choice

1. A. If the match is not cancelled, then it did not rain.
A statement is equivalent to its contrapositive: "if not Q then not P". B is the converse and C the inverse, neither of which follows (the match might be cancelled for another reason); D is the negation.

2. D. 40
$40^2 + 40 + 41 = 1681 = 41^2$, which is not prime. For $n = 1$, 10 and 39 the values 43, 151 and 1601 are all prime. ($n = 41$ also fails, since every term is then divisible by 41.)

3. A. $y = 0$ and $y = \pi$
$\tan^{-1}(u) \to \pm\frac{\pi}{2}$ as $u \to \pm\infty$; adding $\frac{\pi}{2}$ gives 0 and π. The horizontal translation by 1 does not change the asymptotes.

4. B. 1
For $0 \le x < \frac{\pi}{2}$, $\sec(x) \ge 1$ and $\sec(x) > x$. For $\frac{\pi}{2} < x < \frac{3\pi}{2}$, $\sec(x) \le -1 < x$. For $\frac{3\pi}{2} < x \le 2\pi$, $\sec(x)$ falls from $+\infty$ to 1 while x rises past 4.7, so the graphs cross exactly once (near $x \approx 4.92$).

5. C. 8
By de Moivre's theorem, $z^3 = 2^3 \operatorname{cis}\left(3 \times -\frac{2\pi}{3}\right) = 8\operatorname{cis}(-2\pi) = 8$. Cubing triples the argument, and -2π is a full turn.

6. D. i
$w = \frac{(1+i)^2}{(1-i)(1+i)} = \frac{2i}{2} = i$. Since $2025 = 4 \times 506 + 1$, $i^{2025} = i$.

7. A. $1+i,\ 1-i,\ -1+i,\ -1-i$
$-4 = 4\operatorname{cis}(\pi)$, so $z = \sqrt{2}\operatorname{cis}\left(\frac{\pi}{4} + \frac{k\pi}{2}\right) = \pm 1 \pm i$. B has modulus 2, not $\sqrt{2}$; D gives $z^4 = 4$.

8. C. 0.75
Each pass multiplies y by 1.5 ($y \leftarrow y + 0.25 \times 2y$): $y = 1.5, 2.25, 3.375$ at $x = 0.25, 0.5, 0.75$. The loop stops when $y > 3$ and prints $x = 0.75$. The exact solution $y = e^{2x}$ reaches 3 at $x = \frac{\log_e(3)}{2} \approx 0.55$.

9. C. 861
Equilibrium: $0.5P - 0.0005P^2 = 60$, so $P^2 - 1000P + 120\,000 = 0$ and $P \approx 139$ or 861. 1000 is the carrying capacity without harvesting.

10. B. 2
$v\frac{dv}{dx} = -v^2$ gives $\frac{dv}{dx} = -v$, so $v = 4e^{-x}$. At $x = \log_e(2)$, $v = 2$.

11. C. $\dfrac{3\pi}{10}$
$V = \pi\displaystyle\int_0^1 \left((\sqrt{x})^2 - (x^2)^2\right)dx = \pi\left(\frac{1}{2} - \frac{1}{5}\right) = \frac{3\pi}{10}$. $\frac{9\pi}{70}$ squares the difference $(\sqrt{x} - x^2)^2$ instead of subtracting the squares; $\frac{1}{3}$ is the area of the region.

12. D. 5.33
$S = \displaystyle\int_0^1 2\pi x\sqrt{1 + (2x)^2}\,dx = \frac{\pi}{6}\left(5\sqrt{5} - 1\right) \approx 5.33$ (the x-form of the y-axis formula, since $dy = 2x\,dx$). 3.81 rotates about the x-axis instead; 0.85 omits the 2π.

13. B. $\frac{x}{2}\sin(2x) + \frac{1}{4}\cos(2x)$

By parts with $u = x$, $\frac{dv}{dx} = \cos(2x)$: $\frac{x}{2}\sin(2x) - \int \frac{1}{2}\sin(2x)\,dx = \frac{x}{2}\sin(2x) + \frac{1}{4}\cos(2x)$. Check by differentiating. A has the sign error from $\int \sin(2x)\,dx$.

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PREVIEW

That’s the end of the preview answers

These are the answers to the 13 preview questions. The full answer key covers all 26 questions in Specialist Mathematics Unit 3 & 4 — Examination 2 (Practice 5), with the working behind every mark.

STILL TO COME	QUESTIONS	MARKS
Section A — multiple choice	7	7
Section B — extended response	6	60

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- The complete answer key, with the working behind every mark
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