EXAM PAPER

Specialist Mathematics Unit 3 & 4 — Examination 2 (Practice 5)

PREVIEW
This file has the first 13 of the 26 questions in this paper. The rest is locked: it comes with a PrepNest plan, or with the paper itself for VCE.

Reading time: 15 minutes (no writing)
Writing time: 120 minutes
Total time: 135 minutes

SECTIONS IN THIS PAPER

- Section A — multiple choice — 45 min (calculator allowed) — 13 questions, 13 marks
- Section B — extended response — 75 min (calculator allowed) — 0 questions, 0 marks

A formula sheet is printed at the end of this paper.

INSTRUCTIONS

You are not permitted to write during reading time — you may only read the paper and plan your approach. Answer every question you can. Write your working in the space provided for long-answer questions. Marking guidance and full explanations are provided in the separate answer key.

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Section A — multiple choice

45 minutes • Calculator allowed

INSTRUCTIONS

- Answer all questions. Choose the response that is correct for the question.
- A correct answer scores 1; an incorrect answer scores 0. Marks will not be deducted for incorrect answers.
- Unless otherwise indicated, the diagrams in this book are not drawn to scale.
- Take the acceleration due to gravity to have magnitude g m s^{-2}, where g = 9.8.

Question 1

Which one of the following is logically equivalent to the statement ‘If it rains, then the match is cancelled’?

A. If the match is not cancelled, then it did not rain.

B. If the match is cancelled, then it rained.

C. If it does not rain, then the match is not cancelled.

D. It rains and the match is not cancelled.

Question 2

The statement ‘For all $n \in \mathbb{N}$, $n^2 + n + 41$ is prime’ is shown to be false by the counterexample $n =$

A. 1

B. 10

C. 39

D. 40

Question 3

The graph of $y = \tan^{-1}(x - 1) + \dfrac{\pi}{2}$ has horizontal asymptotes with equations

A. $y = 0$ and $y = \pi$

B. $y = -\frac{\pi}{2}$ and $y = \frac{\pi}{2}$

C. $y = 1$ and $y = \pi + 1$

D. $y = 0$ only

Question 4

The number of solutions of the equation $\sec(x) = x$ for $0 \leq x \leq 2\pi$ is

A. 0

B. 1

C. 2

D. 3

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Question 5

If $|z| = 2$ and $\text{Arg}(z) = -\dfrac{2\pi}{3}$, then z^3 is equal to

A. -8
B. $-8i$
C. 8
D. $8i$

Question 6

Let $w = \dfrac{1+i}{1-i}$. Then w^{2025} is equal to

A. 1
B. -1
C. $-i$
D. i

Question 7

The solutions of $z^4 + 4 = 0$, $z \in \mathbb{C}$, are

A. $1+i,\ 1-i,\ -1+i,\ -1-i$
B. $\sqrt{2}+\sqrt{2}i,\ \sqrt{2}-\sqrt{2}i,\ -\sqrt{2}+\sqrt{2}i,\ -\sqrt{2}-\sqrt{2}i$
C. $2,\ -2,\ 2i,\ -2i$
D. $\sqrt{2},\ -\sqrt{2},\ \sqrt{2}i,\ -\sqrt{2}i$

Question 8

The pseudocode below applies Euler's method to a differential equation. The value printed is

```
x ← 0
y ← 1
h ← 0.25
while y ≤ 3 do
    y ← y + h × 2 × y
    x ← x + h
end while
print x
```

A. 0.50
B. 0.55
C. 0.75
D. 1.00

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Question 9

A population with logistic growth is harvested at a constant rate, so that $\frac{dP}{dt} = 0.5P\left(1 - \frac{P}{1000}\right) - 60$. The largest equilibrium population, correct to the nearest whole number, is

A. 139
B. 500
C. 861
D. 1000

Question 10

A particle moves in a straight line with acceleration $a = -v^2$ m s^{-2}, where v m s^{-1} is its velocity when its displacement from the origin is x m. When $x = 0$, $v = 4$. When $x = \log_e(2)$, v is equal to

A. 1
B. 2
C. 3
D. 4

Question 11

The region enclosed by the graphs of $y = \sqrt{x}$ and $y = x^2$ is rotated about the x-axis. The volume of the solid formed is

A. $\frac{1}{3}$
B. $\frac{9\pi}{70}$
C. $\frac{3\pi}{10}$
D. $\frac{\pi}{2}$

Question 12

The curve $y = x^2$, $0 \leq x \leq 1$, is rotated about the y-axis. The area of the curved surface formed, correct to two decimal places, is

A. 0.85
B. 3.14
C. 3.81
D. 5.33

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Question 13

An antiderivative of $x\cos(2x)$ is

A. $\frac{x}{2}\sin(2x) - \frac{1}{4}\cos(2x)$

B. $\frac{x}{2}\sin(2x) + \frac{1}{4}\cos(2x)$

C. $2x\sin(2x) + \cos(2x)$

D. $\frac{x^2}{2}\sin(2x)$

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FORMULA SHEET

Specialist Mathematics formulas

Mensuration	
area of a circle segment	$\frac{r^2}{2}(\theta - \sin(\theta))$
volume of a cylinder	$\pi r^2 h$
volume of a cone	$\frac{1}{3}\pi r^2 h$
volume of a pyramid	$\frac{1}{3}Ah$
volume of a sphere	$\frac{4}{3}\pi r^3$
area of a triangle	$\frac{1}{2}bc\sin(A)$
sine rule	$\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}$
cosine rule	$c^2 = a^2 + b^2 - 2ab\cos(C)$

Algebra, number and structure (complex numbers)	
$z = x + iy = r(\cos(\theta) + i\sin(\theta)) = r\,\mathrm{cis}(\theta)$	$\lvert z\rvert = \sqrt{x^2 + y^2} = r$
$-\pi < \mathrm{Arg}(z) \leq \pi$	$z_1 z_2 = r_1 r_2\,\mathrm{cis}(\theta_1 + \theta_2)$
$\frac{z_1}{z_2} = \frac{r_1}{r_2}\mathrm{cis}(\theta_1 - \theta_2)$	de Moivre's theorem $z^n = r^n\,\mathrm{cis}(n\theta)$

Data analysis, probability and statistics		
for independent random variables $X_1, X_2, \ldots, X_n$	$\mathrm{E}(aX_1 + b) = a\,\mathrm{E}(X_1) + b$	$\mathrm{Var}(aX_1 + b) = a^2\,\mathrm{Var}(X_1)$
	$\mathrm{E}(a_1X_1 + a_2X_2 + \cdots + a_nX_n) = a_1\,\mathrm{E}(X_1) + a_2\,\mathrm{E}(X_2) + \cdots + a_n\,\mathrm{E}(X_n)$	
	$\mathrm{Var}(a_1X_1 + a_2X_2 + \cdots + a_nX_n) = a_1^2\,\mathrm{Var}(X_1) + a_2^2\,\mathrm{Var}(X_2) + \cdots + a_n^2\,\mathrm{Var}(X_n)$	
for independent identically distributed variables $X_1, X_2, \ldots, X_n$	$\mathrm{E}(X_1 + X_2 + \cdots + X_n) = n\mu$	$\mathrm{Var}(X_1 + X_2 + \cdots + X_n) = n\sigma^2$
approximate confidence interval for μ	$\left(\bar{x} - z\frac{s}{\sqrt{n}},\ \bar{x} + z\frac{s}{\sqrt{n}}\right)$	
distribution of sample mean $\bar{X}$	mean $\mathrm{E}(\bar{X}) = \mu$	variance $\mathrm{Var}(\bar{X}) = \frac{\sigma^2}{n}$

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Circular (trigonometric) functions	
$\cos^2(x) + \sin^2(x) = 1$	
$1 + \tan^2(x) = \sec^2(x)$	$\cot^2(x) + 1 = \operatorname{cosec}^2(x)$
$\sin(x+y) = \sin(x)\cos(y) + \cos(x)\sin(y)$	$\sin(x-y) = \sin(x)\cos(y) - \cos(x)\sin(y)$
$\cos(x+y) = \cos(x)\cos(y) - \sin(x)\sin(y)$	$\cos(x-y) = \cos(x)\cos(y) + \sin(x)\sin(y)$
$\tan(x+y) = \dfrac{\tan(x)+\tan(y)}{1-\tan(x)\tan(y)}$	$\tan(x-y) = \dfrac{\tan(x)-\tan(y)}{1+\tan(x)\tan(y)}$
$\cos(2x) = \cos^2(x) - \sin^2(x) = 2\cos^2(x) - 1 = 1 - 2\sin^2(x)$	
$\sin(2x) = 2\sin(x)\cos(x)$	$\tan(2x) = \dfrac{2\tan(x)}{1-\tan^2(x)}$
$\sin^2(ax) = \dfrac{1}{2}(1-\cos(2ax))$	$\cos^2(ax) = \dfrac{1}{2}(1+\cos(2ax))$

Calculus	
$\dfrac{d}{dx}(x^n) = nx^{n-1}$	$\displaystyle\int x^n\,dx = \frac{1}{n+1}x^{n+1} + c,\ n \neq -1$
$\dfrac{d}{dx}(e^{ax}) = ae^{ax}$	$\displaystyle\int e^{ax}\,dx = \frac{1}{a}e^{ax} + c$
$\dfrac{d}{dx}(\log_e(x)) = \dfrac{1}{x}$	$\displaystyle\int \frac{1}{x}\,dx = \log_e\lvert x\rvert + c$
$\dfrac{d}{dx}(\sin(ax)) = a\cos(ax)$	$\displaystyle\int \sin(ax)\,dx = -\frac{1}{a}\cos(ax) + c$
$\dfrac{d}{dx}(\cos(ax)) = -a\sin(ax)$	$\displaystyle\int \cos(ax)\,dx = \frac{1}{a}\sin(ax) + c$
$\dfrac{d}{dx}(\tan(ax)) = a\sec^2(ax)$	$\displaystyle\int \sec^2(ax)\,dx = \frac{1}{a}\tan(ax) + c$
$\dfrac{d}{dx}(\cot(ax)) = -a\operatorname{cosec}^2(ax)$	$\displaystyle\int \operatorname{cosec}^2(ax)\,dx = -\frac{1}{a}\cot(ax) + c$
$\dfrac{d}{dx}(\sec(ax)) = a\sec(ax)\tan(ax)$	$\displaystyle\int \sec(ax)\tan(ax)\,dx = \frac{1}{a}\sec(ax) + c$
$\dfrac{d}{dx}(\operatorname{cosec}(ax)) = -a\operatorname{cosec}(ax)\cot(ax)$	$\displaystyle\int \operatorname{cosec}(ax)\cot(ax)\,dx = -\frac{1}{a}\operatorname{cosec}(ax) + c$
$\dfrac{d}{dx}\left(\sin^{-1}(ax)\right) = \dfrac{a}{\sqrt{1-(ax)^2}}$	$\displaystyle\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\left(\frac{x}{a}\right) + c,\ a > 0$
$\dfrac{d}{dx}\left(\cos^{-1}(ax)\right) = \dfrac{-a}{\sqrt{1-(ax)^2}}$	$\displaystyle\int \frac{-1}{\sqrt{a^2-x^2}}\,dx = \cos^{-1}\left(\frac{x}{a}\right) + c,\ a > 0$
$\dfrac{d}{dx}\left(\tan^{-1}(ax)\right) = \dfrac{a}{1+(ax)^2}$	$\displaystyle\int \frac{a}{a^2+x^2}\,dx = \tan^{-1}\left(\frac{x}{a}\right) + c$
$\displaystyle\int (ax+b)^{-1}\,dx = \frac{1}{a}\log_e\lvert ax+b\rvert + c$	$\displaystyle\int (ax+b)^n\,dx = \frac{1}{a(n+1)}(ax+b)^{n+1} + c,\ n \neq -1$

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Calculus — continued	
product rule	$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$
quotient rule	$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$
chain rule	$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$
integration by parts	$\int u\frac{dv}{dx}\,dx = uv - \int v\frac{du}{dx}\,dx$
Euler's method	If $\frac{dy}{dx} = f(x, y)$, $x_0 = a$ and $y_0 = b$, then $x_{n+1} = x_n + h$ and $y_{n+1} = y_n + hf(x_n, y_n)$.
arc length parametric	$\int_{t_1}^{t_2} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt$
surface area Cartesian about the x-axis	$\int_{x_1}^{x_2} 2\pi y\sqrt{1 + \left(\frac{dy}{dx}\right)^2}\,dx$
surface area Cartesian about the y-axis	$\int_{y_1}^{y_2} 2\pi x\sqrt{1 + \left(\frac{dx}{dy}\right)^2}\,dy$
surface area parametric about the x-axis	$\int_{t_1}^{t_2} 2\pi y\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt$
surface area parametric about the y-axis	$\int_{t_1}^{t_2} 2\pi x\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt$

Kinematics		
acceleration	$a = \frac{d^2x}{dt^2} = \frac{dv}{dt} = v\frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$	
constant acceleration formulas	$v = u + at$	$s = ut + \frac{1}{2}at^2$
	$v^2 = u^2 + 2as$	$s = \frac{1}{2}(u + v)t$

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Vectors in two and three dimensions		
	$\underset{\sim}{r}(t) = x(t)\underset{\sim}{i} + y(t)\underset{\sim}{j} + z(t)\underset{\sim}{k}$	$\lvert\underset{\sim}{r}(t)\rvert = \sqrt{x(t)^2 + y(t)^2 + z(t)^2}$
	$\dot{\underset{\sim}{r}}(t) = \frac{d\underset{\sim}{r}}{dt} = \frac{dx}{dt}\underset{\sim}{i} + \frac{dy}{dt}\underset{\sim}{j} + \frac{dz}{dt}\underset{\sim}{k}$	
for $\underset{\sim}{r}_1 = x_1\underset{\sim}{i} + y_1\underset{\sim}{j} + z_1\underset{\sim}{k}$ and $\underset{\sim}{r}_2 = x_2\underset{\sim}{i} + y_2\underset{\sim}{j} + z_2\underset{\sim}{k}$	vector scalar product $\underset{\sim}{r}_1 \cdot \underset{\sim}{r}_2 = \lvert\underset{\sim}{r}_1\rvert\lvert\underset{\sim}{r}_2\rvert\cos(\theta) = x_1x_2 + y_1y_2 + z_1z_2$	
	vector cross product $\underset{\sim}{r}_1 \times \underset{\sim}{r}_2 = \begin{vmatrix} \underset{\sim}{i} & \underset{\sim}{j} & \underset{\sim}{k} \\ x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \end{vmatrix} = (y_1z_2 - y_2z_1)\underset{\sim}{i} + (x_2z_1 - x_1z_2)\underset{\sim}{j} + (x_1y_2 - x_2y_1)\underset{\sim}{k}$	
vector equation of a line	$\underset{\sim}{r}(t) = \underset{\sim}{r}_1 + t\underset{\sim}{r}_2 = (x_1 + x_2t)\underset{\sim}{i} + (y_1 + y_2t)\underset{\sim}{j} + (z_1 + z_2t)\underset{\sim}{k}$	
parametric equation of a line	$x(t) = x_1 + x_2t \quad y(t) = y_1 + y_2t \quad z(t) = z_1 + z_2t$	
vector equation of a plane	$\underset{\sim}{r}(s,t) = \underset{\sim}{r}_0 + s\underset{\sim}{r}_1 + t\underset{\sim}{r}_2 = (x_0 + x_1s + x_2t)\underset{\sim}{i} + (y_0 + y_1s + y_2t)\underset{\sim}{j} + (z_0 + z_1s + z_2t)\underset{\sim}{k}$	
parametric equation of a plane	$x(s,t) = x_0 + x_1s + x_2t \quad y(s,t) = y_0 + y_1s + y_2t \quad z(s,t) = z_0 + z_1s + z_2t$	
Cartesian equation of a plane	$ax + by + cz = d$	

END OF FORMULA SHEET

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6.

13 questions locked, up to question 6.

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PREVIEW

That’s the end of the preview

You’ve seen the first 13 of the 26 questions in Specialist Mathematics Unit 3 & 4 — Examination 2 (Practice 5). The full paper is worth 80 marks.

STILL TO COME	QUESTIONS	MARKS
Section A — multiple choice	7	7
Section B — extended response	6	60

The full paper comes with

- Every question, printed exactly like the ones in this preview
- The complete answer key, with the working behind every mark
- A topic report after marking, showing what to practice next

Get the full paper at
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